optimization equalizes marginal rates of substitution accross agents:

$$\frac{\partial u/\partial c_t^t}{\partial u/\partial c_{t+1}^t} = 1 + r \quad \text{for all } t$$

However, because of the OLG setup, nothing guarantees that the competitive equilibrium achieves the welfare optimum, i.e. that $r = n$ and therefore $k = k^*$. In particular, it is possible that $r < n$, in which case the competitive equilibrium is dynamically inefficient. Since the capital stock is above the golden-rule level, a Pareto improvement is possible in the following way: The currently old generation consumes the excess capital stock, making them better off; all future generations have to save less and can consume more, making them better off as well.[17]

As a solution to this potential inefficiency, Diamond (1965) proposes the use of government debt with a constant per capita level d, issued at the market interest rate r. The effect of this intervention is that it crowds out investment – since part of the young generation's saving now goes into purchasing bonds instead of capital – and raises the interest rate r, thus shrinking the inefficiency gap $n - r$.

Bubbles. Tirole (1985) uses the same framework as Diamond (1965) with capital and production but instead of government debt, he studies the effect of rational bubbles. As in the original paper by Samuelson, he introduces an asset that cannot be used for consumption or production but trades at price b_t. With rational investors the asset price has to satisfy $b_t \geq 0$ and

$$b_{t+1} = \frac{1 + r_{t+1}}{1 + n} b_t.$$

Just like the government bonds in Diamond (1965), the bubble asset uses up a part of savings, crowding out productive investment and increasing the interest rate. Therefore, if the baseline economy is dynamically inefficient, there is a steady state with a bubble $b > 0$ that achieves the welfare optimum $r = n$.[18] In addition, a bubble

[17] Blanchard (1985) studies a "perpetual-youth model" where agents have a constant probability of dying in each period and therefore a constant finite expected horizon. Compared to an infinite-horizon model, the finite horizon reduces the incentive to save, decreasing capital accumulation. Adding labor income that decreases with age increases the incentive to save and the steady state can be inefficient as in the present OLG setting.

[18] However, the equilibrium path leading to the steady state is only saddle-path stable. This means that only one initial bubble size results in the efficient steady state while the paths for all other initial bubbles have $b_t \to 0$, resulting in the baseline steady state.