

endowment which obviously clears markets so there is a second competitive equilibrium which implements an autarky allocation. This autarkic competitive equilibrium is clearly Pareto inefficient, even though markets are complete.¹⁶ The underlying cause is of this potential for inefficiency which doesn't exist in an Arrow-Debreu setting can be thought of as a "lack of market clearing at infinity." See [Geanakoplos \(2008\)](#) for a detailed discussion of the technical details.

In the original paper, [Samuelson \(1958\)](#) focuses on equilibria that can be implemented in a sequential exchange economy. Therefore, in the basic version of the model with only the perishable consumption good, the only achievable competitive equilibrium is the autarky equilibrium. However, things change substantially with the introduction of a durable asset that provides a store of value. Even though this asset cannot be used for consumption, now the Pareto optimal allocation is attainable as a competitive equilibrium. In this equilibrium the asset, e.g. fiat money, trades at a price b_t which grows at the same rate as the population:

$$b_{t+1} = (1 + n) b_t$$

By transferring wealth *within* a period from the young generation to the old generation, the asset allows to transfer wealth *across* periods from the youth of a generation to their own old age.

Production. [Diamond \(1965\)](#) uses the same setup as [Samuelson \(1958\)](#) but adds a capital good which, together with labor, is used to produce the consumption good with a constant-returns-to-scale aggregate production function $Y_t = F(K_t, L_t)$. The consumption good can be converted into new capital one-for-one and capital doesn't depreciate.

The welfare-optimal steady state requires, as before, that the marginal rates of substitution are equalized across all agents in all periods and that they are equal to the growth rate $1 + n$. In addition, the steady state capital stock has to maximize production subject to the aggregate budget constraint. Denoting per-capita values by lowercase letters, this implies that the optimal level of the capital stock to satisfies $f'(k^*) = n$, which is commonly known as the "golden rule."

In the competitive equilibrium, capital is paid a rental rate $r = f'(k)$ and individual

¹⁶In addition, there is an infinite number of non-stationary competitive equilibria, i.e. with time-changing interest rate r_t that are also Pareto inefficient.