4.1 Smoothing Deterministic Fluctuations

Basic OLG. Models of overlapping generations (OLG) are used to analyze the role of liquid assets to improve allocations. Many of the economic insights also arise in an incomplete markets setting discussed below. While the initial OLG models took the interpretation of generations literally, more recent papers use it as a tractable short-cut formualtion for other financial frictions. Of course, the latter "renders" a quantitative evaluation and calibration.

The concept of finitely-lived but overlapping generations is first introduced in Samuelson (1958). The paper models an infinite-horizon economy where in each period t, a new generation of agents is born who live for two periods. An agent in generation t therefore only derives utility from consumption in periods t and $t+1$, i.e. his utility is given by $u\left(c_t^t, c_{t+1}^t\right)$. The size of each new generation and therefore the entire population grows at a rate n.

In this setting, a Pareto optimal allocation requires that the marginal rates of intertemporal substitution are equalized across all agents and that they are equal to the population growth rate,

$$\frac{\partial u / \partial c_t^t}{\partial u / \partial c_{t+1}^t} = 1 + n \quad \text{for all } t.$$

The peculiar feature of the OLG structure as opposed to a standard Arrow-Debreu setting is that even with complete markets – that is, even if all generations could meet at time $t = 0$ and write contingent contracts – OLG economies can have multiple competitive equilibria that can be Pareto ranked.

Consider the following simple example. Let the utility function be given by

$$u\left(c_t^t, c_{t+1}^t\right) = \ln\left(c_t^t\right) + \beta \ln\left(c_{t+1}^t\right)$$

and let each generation have an endowment e when young and $1 - e$ when old. In addition, assume that markets are complete, i.e. agents can borrow and lend freely at an interest rate r. The first order conditions of an agent in generation t imply

$$\frac{c_{t+1}^t}{\beta c_t^t} = 1 + r$$

and there is a competitive equilibrium with $1 + r = 1 + n$ that implements the Pareto optimum.

However, note that for $1 + r = (1 - e) / (\beta e)$ each agent simply consumes his