

after a negative shock. This is in sharp contrast to [Brunnermeier and Pedersen \(2009\)](#) in which the volatility dynamics and the resulting margin/haircut spiral forces experts to delever in times of crisis. To see this formally, focus on the second and third term in the denominator of

$$\frac{\partial q_1}{\partial \eta_1} = \frac{1}{\frac{2}{\gamma(\sigma_2)^2} m_1^+ - x_0 + \frac{\partial m_1^+}{\partial q_1} x_1}.$$

If experts hold a positive position of this asset, i.e.  $x_0 > 0$ , then losses amplify the price impact (loss spiral). Furthermore, if a decline in price, leads to higher margins/haircuts, i.e.  $\frac{\partial m_1^+}{\partial q_1} < 0$ , experts are forced to delever which destabilizes the system further (margin/haircut spiral). Fragility and margin spiral describe a “*collateral run*” in the ABCP and Repo market in 2008. Collateral runs are the modern form of bank runs and differ from the classic “*counterparty run*” on a particular bank. We will study “counterparty runs” in Section 5 when we discuss [Diamond and Dybvig \(1983\)](#)

In a setting with multiple assets, asset prices might comove even though their cash flows are independently distributed since they are exposed to the same funding liquidity constraint. Also, assets with different margin constraints, might trade at vastly different prices even when their payoffs are similar. See also [Gârleanu and Pedersen \(2011\)](#).

### 3.3 Equilibrium Margins and Endogenous Incompleteness

[Geanakoplos \(1997, 2003\)](#) studies endogenous collateral/margin constraints in a general equilibrium framework à la Arrow-Debreu. Unlike in an Arrow-Debreu world, in Geanakoplos’ “collateral equilibrium” no payments in future periods/states can be credibly promised unless they are 100% collateralized with the value of durable assets. With the effect of asset prices on borrowing, Geanakoplos’ collateral constraint is similar to the one in KM97, but here collateralized borrowing, equilibrium margins/haircuts are derived endogenously in interaction with equilibrium prices. An important consequence is that markets can be endogenously incomplete.

**Collateral Equilibrium** Consider the following simplified setup. There are two periods  $t = 0, 1$ , and a finite set of states  $s \in S$  in  $t = 1$ . Commodities are indexed by  $\ell \in L$  and some of these are durable between periods 0 and 1 and/or yield output in the form of other commodities in period 1. The potential for durability and transformation is given exogenously by a linear function  $f$ , where a vector  $x$  of goods in period 0 is transformed into a vector  $f_s(x)$  of goods in state  $s$  in period 1.