to

$$dk_t = (\Phi(\iota_t) - \delta)k_t \, dt + \sigma k_t \, dZ_t \tag{18}$$

where $\iota_t k_t$ is the investment rate (i.e. ι_t is the investment rate per unit of capital), the concave function $\Phi(\iota_t)$ reflects (dis)investment costs as in BGG. As before, we refer to $(\Phi(\iota_t) - \delta)$ as technological illiquidity. Households do not invest and hence the law of motion of $\underline{k}_t$ when managed by households is

$$d\underline{k}_t = -\underline{\delta} \underline{k}_t \, dt + \sigma \underline{k}_t \, dZ_t. \tag{19}$$

Note that instead of TFP shocks on a, in BruSan10 capital is shocked directly through Brownian shocks dZ_t. This formulation preserves scale invariance in aggregate capital K_t and can also be expressed as TFP shocks. However, it requires capital to be measured in efficiency units rather than physical number of machines. That is, efficiency losses are interpreted as declines in K_t.

Both experts and less productive households are assumed to be risk neutral. Experts discount future consumption at the rate ρ and their consumption has to be non-negative. On the other hand, less productive households have a utility discount rate of $r < \rho$.[10] Since their consumption need not necessarily be positive, the risk free rate is always equal to r.

There is a fully liquid market for physical capital, in which experts can trade capital among each other or with households. Denote the market price of capital (per efficiency unit) in terms of output by q_t and its law of motion by

$$dq_t = \mu_t^q q_t \, dt + \sigma_t^q q_t \, dZ_t.$$

In equilibrium q_t with its drift μ_t^q and volatility σ_t^q is determined *endogenously* through supply and demand relationships. The total risk of the value of capital $k_t q_t$ consists of the exogenous risk summarized by σ of Equations (18) and (19) and the endogenous price risk captured by σ_t^q. Note that the *endogenous risk* is time-varying and depends on the wealth of the experts.

To solve for the equilibrium, it is instructive to first focus on the less productive households. Since they are risk-neutral and their consumption is unrestricted, their

[10]Like in CF and KM97 the difference in the discount rates ensure that the experts do not accumulate so much wealth such that they do not need additional funding. Recall that in BGG this is achieved by assuming that experts die at a certain rate and consume just prior to death.