be of the form ωR^k_{t+1}, where R^k_{t+1} is the endogenous aggregate equilibrium return and ω is an idiosyncratic shock, i.i.d. across entrepreneurs with $E[\omega] = 1$ and c.d.f. $G(\omega)$.

As before, entrepreneurs borrow from households via debt in a costly state verification framework. Verification costs are a fraction $\mu \in (0,1)$ of the amount extracted from entrepreneurs. For a benchmark scenario when R^k_{t+1} is deterministic, verification occurs when $\omega < \bar{\omega}$ such that households break even

$$\left[(1-\mu)\int_0^{\bar{\omega}} \omega dG(\omega) + (1-G(\bar{\omega}))\,\bar{\omega}\right] R^k_{t+1} q_t k_{t+1} = R_{t+1}\left(q_t k_{t+1} - n_t\right), \tag{8}$$

where R_{t+1} is the risk-free rate.

If there is aggregate risk in R^k_{t+1}, then BGG appeal to their assumption that entrepreneurs are risk-neutral and households are risk-averse to argue that entrepreneurs insure risk-averse households against aggregate risk.[3] If so, then equation (8) has to determine $\bar{\omega}$ as a function of R^k_{t+1} state by state. As in CF, since households can finance multiple entrepreneurs, they can perfectly diversify entrepreneur idiosyncratic risk.

BGG assume that entrepreneurs simply maximize their net worth in the next period, putting off consumption until a later date.[4] As a result, entrepreneurs simply solve

$$\max_{k_{t+1}} E\left[\int_{\bar{\omega}}^{\infty} (\omega - \bar{\omega})\, dG(\omega)\, R^k_{t+1} q_t k_{t+1}\right], \tag{9}$$

subject to the financing constraint (8), which determines how $\bar{\omega}$ depends on R^k_{t+1}.

In equilibrium, the optimal leverage of entrepreneurs depends on their expected return on capital $E\left[R^k_{t+1}\right]$. In fact, entrepreneur optimal leverage is again given by a linear rule

$$q_t k_{t+1} = \psi\left(\frac{E\left[R^k_{t+1}\right]}{R_{t+1}}\right) n_t. \tag{10}$$

[3]Note that these contracts with perfect insurance are not optimal. More generally, the optimal cutoff $\bar{\omega}$ as a function of R^k_{t+1} depends on the trade-off between providing households with better insurance against aggregate shocks, and minimizing expected verification costs. According to the costly state verification framework, the marginal cost of extracting an extra dollar from the entrepreneur is independent of the realization of aggregate return R^k_{t+1}. Therefore, if both entrepreneurs and households were risk-neutral, the optimal solution to the costly state verification problem would set $\bar{\omega}$ to the same value across all realizations of aggregate uncertainty, i.e. aggregate risks would be shared proportionately between the two groups of agents. See Gale and Hellwig (1985) for an early example that a standard debt contracts is no longer optimal when the entrepreneur is risk averse.

[4]To prevent entrepreneurs from accumulating infinite wealth, this requires the additional assumption that each entrepreneur dies with a certain probability each period in which case he is forced to consume his wealth and is replaced by a new entrepreneur.