

(n - 1) restrictions, $i = 1, \dots, (n - 1)$

$$P \cdot \gamma - Q = 0 \tag{119}$$

This does not involve estimating the restricted nonlinear model. My own results on British data, Deaton (1981b), suggest relatively little conflict with separability, however, earlier work by Atkinson and Stern (1981) on the same data but using an ingenious adaptation of Becker's (1965) time allocation model, suggests the opposite. Blundell and Walker (1982), using a variant of (117) reject the hypothesis that wife's leisure is separable from goods. Separability between different time periods is much more difficult to test since it is virtually impossible to provide general unrestricted estimates of the substitution responses between individual commodities across different time periods.

Subgroup demand functions are only a part of what the applied econometrician needs from separability. Just as important is the question of whether it is possible to justify demand functions for commodity composites in terms of total expenditure and composite price indices. The Hicks (1936) composite commodity theorem allows this, but only at the price of assuming that there are no relative price changes within subgroups. Since there is no way of guaranteeing this, nor often even of checking it, more general conditions are clearly desirable. In fact, the separable structure (114) may be sufficient in many circumstances. Write u_A, u_P , etc. for the values of the felicity functions and $c_A(u_A, P^A)$ etc. for the subgroup cost functions corresponding to the $v_A(q^A)$ functions. Then the problem of choosing the group expenditure levels x, x_P, \dots can be written as

$$\begin{aligned} \max u = & \tag{120} \\ \text{s.t. } x = & E c_A(u_P, p^R). \end{aligned}$$

Write

$$c_R(u_R, p^R) = \frac{c_R(u_R, p^R)}{c_R(u_R, p^R)} \tag{121}$$

for some fixed prices p^R . For such a fixed vector, $c_P(u_P, p^R)$ is a welfare indicator or quantity index, while the ratio $c_R(u_R, p^R)/c_R(u_R, p^R)$ is a true (sub) cost-of-living price index comparing p^R and p^R using u_P as reference, see Pollak (1975). Finally, since $u_R = R(c_R(u_R, p^R), p^R)$, (120) may be written

$$\begin{aligned} \max u = & (1, 1) A(c_A(u_A, P^A), J_i^A), A d_B(\cdot), \tag{122} \\ \text{s.t. } E c_R(u_R, p^R) & \cdot c_R(u_R, p^R) \end{aligned}$$