

maximization of overall u implies maximization of the subutilities subject to whatever is optimally spent on the groups. Hence, (113) implies the existence of subgroup demands

$$S_{A,p} \quad (115)$$

where $x^A = p^A \cdot q^A$, while (115) has the same implication for all groups. Hence, if preferences in a life-cycle model are weakly separable over time periods, commodity demand functions conditional on x and p for each time period are guaranteed to exist. Similarly, if goods are separable from leisure, commodity demand functions of the usual type can be justified.

Tests of these forms of separability can be based on the restrictions on the substitution matrix implied by (115). If i and j are two goods in distinct groups, $G, j \in H, G \neq j$, then the condition

$$\frac{dq_i}{dx_j} = \frac{dq_j}{dx_i}$$

for some quantity p_{Gij} (independent of i and j) is both necessary and sufficient for (114) to hold. If a general enough model of substitution can be estimated, (116) can be used to test separability, and Byron (1968), Jorgenson and Lau (1975) and Pudney (1981b), have used essentially this technique to find separability patterns between goods within a single period. Barnett (1979a) has tested the important separability restriction between goods and leisure using time series American data and decisively rejects it. If widely repeated, this result would suggest considerable misspecification in the traditional studies. It is also possible to use a single cross-section to test separability between goods and leisure. Consider the following cost function proposed by Muellbauer (1981b).

$$c(u, w, p) = d(p) + b(p)k + (a(p))^{1-\delta} w^\delta u, \quad (117)$$

where w is the wage p , $b(p)$ and $a(p)$ are functions of p , homogenous of degrees, 1, 0 and 1 respectively. Shephard's Lemma gives immediately

$$\frac{\partial c}{\partial h} = a + \frac{1}{3} \frac{\partial a}{\partial p} + \frac{1}{4} \frac{\partial a}{\partial p} \quad (118)$$

for transfer income p , hours worked h and parameters a, β, γ all constant in a single cross-section. It may be shown that (117) satisfies (114) for leisure vis-à-vis goods if and only if $h(p)$ is a constant, which for (118) implies that $h(p)$ be independent of $i, i = 1, \dots, n$. This can be tested by first estimating (114) as a system by OLS equation by equation and then computing the Wald test for the

