

markets. Housing is the obvious example, but here I illustrate with a simple case based on Deaton (1984). In many developing countries, the government operates so-called "fair price" shops in which certain commodities, e.g. sugar or rice, are made available in limited quantities at subsidized prices. Typically, consumers can buy more than the fair price allocation in the free market at a price p_1 , with $p_1 > P_0$ the fair price price. Figure 2 illustrates for "sugar" versus a numeraire good with unit price. Z is the amount available in the fair price shop and the budget constraint assumes that resale of surplus at free market prices is impossible.

There are two interrelated issues here for empirical modelling. At the micro level, using cross-section data, we need to know how to use utility theory to generate Engel curves. At the macro-level, it is important to know how the two prices p_0 and p_1 and the quantity Z affect total demand. As usual, we begin with the indirect utility function, though the form of this can be dictated by prior beliefs about demands (e.g. there has been heavy use of the indirect utility function associated with a linear demand function for a single good—for the derivation, see Deaton and Muellbauer (1980a, p. 96) (1981) and Hausman

(1980)). Maximum utility along AD is $u_0(x, p, p_0)$ with associated demand, by Roy's identity, of $s_0 = g(x, p, p_0)$. Now, by standard revealed preference, if $s_0 < Z$, s_0 is optimal since BC is obtainable by a consumer restricted to being within AD. Similar, maximum utility along EC is $u_1(x + (P_1 - P_0)Z, P, P_1)$ with $s_1 = g(x + (p_1 - p_0)Z, p, p_1)$. Again, if $s_1 > Z$, then s_1 is optimal. The remaining case is $s_0 > Z$ and $s_1 < Z$ (both of which are infeasible), so that sugar demand is exactly Z (at the kink B). Hence, for individual h with expenditure x^h and quota Z^h , the demand functions are given by

$$s^h = g^h(x^h, p, p_0) \quad \text{if } g^h(x^h, p, p_0) < Z^h \tag{107}$$

$$s^h = g^h(x^h + (p_1 - p_0)Z^h, p, p_1) \quad \text{if } g^h(x^h + (p_1 - p_0)Z^h, p, p_1) > Z^h \tag{108}$$

$$s^h = Z^h \quad \text{if } g^h(x^h - (P_1 - P_0)Z^h, p, p_1) < Z^h < g^h(x^h, p, p_0) \tag{109}$$

Figure 3 gives the resulting Engel curve. Estimation on cross-section data is straightforward by an extension of the Tobit method; the demand functions g^h are endowed with taste variation in the form of a normally distributed random term, and a likelihood with three "branches" corresponding to $s^h < Z^h$, $s^h = Z^h$, and $s^h > Z^h$ is constructed. The middle branch corresponds to the zero censoring for Tobit; the outer two are analogous to the non-censored observations in Tobit.

The aggregate free-market demand for sugar can also be analysed using the model. To simplify, assume that households differ only in outlay, x^h . Define x_T by $g(x_T + p_1 - p_0)Z, p, p_1) = Z$, so that consumers with $x > x_T$ enter the free