

to be theory consistent, there must exist a cost function $c(\mathbf{u}, \mathbf{p})$ such that

$$\frac{d \ln c(\mathbf{u}, \mathbf{p})}{d \mathbf{p}}$$

$$= \sum_{i=1}^I \frac{a_i(\mathbf{p})}{c(\mathbf{u}, \mathbf{p})} \frac{d c(\mathbf{u}, \mathbf{p})}{d \mathbf{p}} \quad (76)$$

Gorman shows that for these partial differential equations to have a solution, (a) the rank of the matrix formed from the coefficients $a_i(\mathbf{p})$ can be no larger than 3 and (b), the functions $a_i(\mathbf{p})$ must take specific restricted forms. There are three generic forms for (75), two of which are reproduced below

$$a_i(\mathbf{p}) = \sum_{m=1}^M \beta_m(\mathbf{p}) \ln x_i \quad (77)$$

$$a_i(\mathbf{p}) = \beta_i(\mathbf{p}) x_i^{-\alpha} \quad (78)$$

where \mathbf{S} is a finite set of elements \mathbf{o} , \mathbf{S}_- its negative elements and \mathbf{S}_+ its positive elements. A third form allows combinations of trigonometrical functions of \mathbf{x} capable of approximating a quite general function of \mathbf{x} . However, note that the β_m and O_m functions in (77) and (78) are *not* indexed on the commodity subscript i , otherwise the rank condition on a_i , could not hold.

Equations (77) and (78) provide a rich source of Engel curve specifications and contain as special cases a number of important forms. From (77), with $\mathbf{in} = \mathbf{1}$, the form proposed by Working and Leser and discussed above, see (15), is obtained. In econometric specifications, $a_i(\mathbf{p})$ adds to unity and $b_i(\mathbf{p})$ to zero, as will their estimates if OLS is applied to each equation separately. The log quadratic form

$$= a_i(\mathbf{p}) - F b_i(\mathbf{p}) \ln x + d_i(\mathbf{p}) (\ln x)^2, \quad (79)$$

was applied in Deaton (1981c) to Sri Lankan micro household data for the food share where the quadratic term was highly significant and a very satisfactory fit was obtained (an R^2 of 0.502 on more than 3,000 observations.) Note that, while for a single commodity, higher powers of $\ln x$ could be added, doing so in a complete system would require cross-equation restrictions since, according to (77), the ratios of coefficients on powers beyond unity should be the same for all commodities. Testing such restrictions (and Wald tests offer a very simple method-see Section 4(a) below) provides yet another possible way of testing the theory.

Equation (78) together with $\mathbf{S} = -\mathbf{1}, \mathbf{1}, 2, \dots, \mathbf{r}, \dots$ gives general polynomial Engel curves. Because of the rank condition, the quadratic with $\mathbf{S} = (-\mathbf{1}, \mathbf{1}, \mathbf{1})$ is as