total of (2 + *n)(n -1)* parameters -(n - 1) a 's and /3's, and *n(n -1) y's* -or (n + 2) per equation as in the previous example. But now, each equation has 2+ (n - 1)n parameters since all y 's always appear. In consequence, if the constant, In x, in p, and the cross-terms are linearly independent in the sample, and if *T* < 2+ (n -1)n, it is possible to choose parameters such that the calculated residuals for any one (arbitrarily chosen) equation will be exactly zero for all sample points. For these parameters, one row and one column of the estimated *SI* will also be zero, its determinant will be zero and the log likelihood (41) or (43) will be infinite. Hence full information MLE's do not exist. In such a case, at least 56 observations would be necessary to estimate an 8 commodity disaggregation. All these cases are variants of the familiar " undersized sample" problem in FIML estimation of simultaneous equation systems and they set upper limits to the amount of commodity disaggregation that can be countenanced on any given time-series data.

Given a singular variance-covariance matrix, for whatever reason, the log likelihood (41) which contains the term - T/2 log det D, will be infinitely large and FIML estimates do not exist. Nor, in general, can (47) be used to calculate GLS or SURE estimators if a singular estimate of 52 is employed. However, there are a number of important special cases in which (47) has solutions that can be evaluated even when *12* is singular (though it is less than clear what is the status of these estimators). For example, in the classical multivariate regression model (49), the solution to (47) is the GLS matrix estimator *B-(1CX)* 1X'Y which does not involve 0, see e.g. Goldberger (1964, pp. 207-42). Imposing identical *within equation* restrictions on (49), e.g. homogeneity, produces another (restricted) classical model with the same property. With *cross-equation* restrictions of the form *R/3 = r,* e.g. symmetry, for stacked /3, /3, the solution to (47) is

$$+ (Sie(X/X)^{-1}\}R1R\{S20(rX)^{-1}\}R'l\ ^{l}(r - Rij), \qquad (52)$$

which, though involving 12, can still be calculated with *0* singular provided the matrix in square brackets is non-singular. I have not been able to find the general conditions on (47) that allow solutions of this form, nor is it clear that it is important to do so. General non-linear systems will not be estimable on undersized samples, and except in the cases given where closed-form solutions exist, attempts to solve (47) and (48) numerically will obviously fail.

The important issue, of course, is the small sample performance of estimators based on near-singular or singular estimates of O. In most time series applications with more than a very few commodities, Q is likely to be a poor estimator of 12 and the introduction of very poor estimates of 52 into the procedure for parameter estimation is likely to give rise to extremely inefficient estimates of the latter. Paradoxically, the search for (asymptotic) efficiency is likely to lead, in this case,