2.2. Singularity of the variance — covariance matrix

The second problem arises from the fact that with x_t *defined* as the sum of expenditures, expenditures automatically add-up to total expenditure identically, i.e. without error. Hence, provided *f,* in (32) is properly chosen, we must have

$$\text{Ep,,}_{q}, x_{t;} \qquad Ef_{i}(p_{,,}x_{f;}\ b) = x_{,;} \qquad \text{Eu,,-o.} \tag{38}$$

Writing *2* as the n x n contemporaneous variance-covariance matrix of the u,,'s with typical element co_{if}, i.e.

$$E(u_{,,}\ u_{p}) \tag{39}$$

then the last part of (38) clearly implies

$$\text{Ewi}_{;} = \text{Ewii=} \tag{40}$$

so that the variance-covariance matrix is singular. If (32) is stacked in the usual way as an *nT* observation regression, its covariance matrix is 001 which cannot have rank higher than (n -1)T. Hence, the usual generalized least squares estimator or its non-linear analogue is not defined since it would require the non-existent inverse 0^{-1}®*I.*

This non-existence is, however, a superficial problem. For a set of equations such as (32) satisfying (38), one equation is essentially redundant and all of its parameters can be inferred from knowledge of those in the other equations. Hence, attempting to estimate all the parameters in all equations is equivalent to including some parameters more than once and leads to exactly the same problems as would arise if, for example, some independent variables were included more than once on the right hand side of an ordinary single-variable regression. The solution is obviously to drop one of the equations and estimate the resulting (n -1) equations by GLS, Zellner's (1962) seemingly unrelated regressions estimator (SURE), or similar technique. Papers by McGuire, Farley, Lucas and Winston (1968) and .by Powell (1969) show that the estimates are invariant to the particular equation which is selected for omission. Barten (1969) also considered the maximum-likelihood estimation of such systems 'when the errors follow the multivariate normal assumption. If f?,, is the variance-covariance matrix of the system (32) excluding the nth equation, a sample of *T* observations has a log-likelihood conditional on normality of

$$\ln L = y(n-1)\text{1n2a} - \text{lndet}\ 12_{,,} - \text{I}\sum_{r=1}^{T} u_{(,,yr}as_{,,}\ u_{(n),,} \tag{41}$$