1.4. Inverse demand functions

In practical applications, it is occasionally necessary to estimate prices as a function of quantities rather than the other way round. An approach to specification exists for this case which is precisely analogous to that suggested above. From the direct utility function and the first-order conditions (10), apply the budget constraint $p.g = x$ to give

$$p_i q_i = du/\ ding,\ x \qquad (24)$$

$$\text{Eatyabi}_{gk}'$$

which is the dual analogue
of (14), though now determination goes from the quantities q to the normalized prices p/x. Alternatively, define the distance function $cl(u,g)$, dual to the cost function, by

$$d(u,g) = \underset{p}{} \{p \bullet g;\ 4'(1,p) \qquad (25)$$

The distance function has properties analogous to the cost function and, in particular,

$$p,/x = cl(u,g)/dg_i = a_i(u,q), \qquad (26)$$

are the inverse compensated demand functions relating an indifference curve u and a quantity ray q to the price to income ratios at the intersection of q and u. See McFadden (1978), Deaton (1979) or Deaton and Muellbauer (1980a, Chapter 2.7) for fuller discussions.

Compensated and uncompensated inverse demand functions can be used in exactly the same way as direct demand functions and are appropriate for the analysis of situations when quantities are predetermined and prices adjust to clear the market. Hybrid situations can also be analysed with some prices fixed and some quantities fixed; again see McFadden (1978) for discussion of "restricted" preference representation functions. Note one final point, however. The Hessian matrix of the distance function $d(u,g)$ is the *Antonelli matrix* A with elements

$$\frac{a2d}{dg,dq_j} \qquad \frac{aa,(u,q)}{dq1} \qquad (27)$$

which can be used to define q-substitutes and q-complements just as the Slutsky matrix defines p-substitutes and p-complements, see Hicks (1956) for the original discussion and derivations. Unsurprisingly the Antonelli and Slutsky matrices are intimately related and given the close parallel been duality and matrix inversion,