which can then be estimated. Of course, the demands corresponding to the original cost function may not fit the data or may have other undesirable properties for the purpose at hand. To build this back into preferences, we must be able to go from *Mx, p)* back to *c(u, p).* But, from Shephard's Lemma, $q_i = g_i(x, p)$ may be rewritten as

(12)

$$\frac{\partial c(u, p)}{\partial p_i} = g_i(c(u, p), P),$$

which may be solved for *c(u, p)*
provided the mathematical integrability conditions are satisfied. These turn out to be equivalent to Slutsky symmetry, so that demand functions displaying symmetry always imply some cost function, see, for example, Hurwicz and Uzawa (1971) for further details. If the Slutsky matrix is also negative semi-definite (together with symmetry, the 'economic' integrability condition), the cost function will be appropriately concave which it must be to represent preferences. This possibility, of moving relatively easily between preferences and demands, is of vital importance if empirical knowledge is to be linked to economic theory.

An alternative and almost equally straightforward procedure is to start from the indirect utility function *11/(x, p).* This must be zero degree homogeneous in *x* and *p* and quasi-convex in *p* and Shephard's Lemma takes the form

$$q\, g,(x, p) - \frac{-\partial 0x, P)/aPi}{191 P(x, P)/\, dx} \tag{13}$$

a formula known as *Roy's identity,* Roy (1942). This is sometimes done in "normalized" form. Clearly, $4_i(x, p) = p/x) = 4^*(r)$ where $r = p/x$ is the vector of normalized prices. Hence, using 4,* instead of tp, Roy's identity can be written in the convenient form

where the last equality follows from rewriting (9)

One of the earliest and best practical examples of the use of these techniques is Samuelson's (1947-8) derivation of the utility function (5) from the specification of the linear expenditure system suggested earlier by Klein and Rubin (1947-8). A more recent example is provided by the following. in 1943, Holbrook Working suggested that a useful form of Engel curve was given by expressing the budget share of good i, w,, as a linear function of the logarithm of total expenditure.