If x is the total budget to be allocated, then x will be the cheapest way of reaching whatever u can be reached at p and x, so that

$$c(u, p) = x. \tag{8}$$

The function $c(u, p)$ can be shown to be continuous in both its arguments, monotone increasing in u and monotone non-decreasing in p. *It* is linearly homogeneous and *concave* in prices, and first and second differentiable almost everywhere. It is *strictly* quasi-concave if *u(q) is differentiable* and *everywhere* differentiable if $u(q)$ is *strictly* quasi-concave. For proofs and further discussions see McFadden (1978), Diewert (1974a), (1980b) or, less rigorously, Deaton and Muellbauer (1980a, Chapter 2).

The empirical importance of the cost function lies in two features. The first is the 'derivative property', often known as Shephard's Lemma, Shephard (1953). By this, whenever the derivative exists

$$\frac{\partial c(u, p)}{\partial p_i} = h_i(u, p) = q_i. \tag{9}$$

The functions $h_i(u, p)$ are known as *Hicksian* demands, in contrast to the *Marshallian* demands $g_i(x, p)$. The second feature is the Shephard—Uzawa duality theorem [again see McFadden (1978) or Diewert (1974a), (1980b)] which given convex preferences, allows a constructive recovery of the utility function from the cost function. Hence, all the information in $v(q)$ which is relevent to behavior and empirical analysis is encoded in the function $c(u, p)$. Or put another way, any function $c(u, p)$ with the correct properties can serve as an alternative to $v(q)$ *as a basis* for empirical analysis. The direct utility function need never be explicitly evaluated or derived; if the cost function is correctly specified, corresponding preferences always exist. The following procedure is thus suggested in empirical work. Starting from some linearly homogeneous concave cost function $c(u, p)$, derive the Hicksian demand functions $h_i(u, p)$ by differentiation. These can be converted into Marshallian demands by substituting for u from the inverted form of (8); this is written

$$u = \psi(x, p), \tag{10}$$

$$q_i = h_i(u, p) = h_i(\psi(x, p), p) = g_i(x, p), \tag{11}$$

and is known as the *indirect* utility function. (The original function $v(q)$ is the *direct* utility function and the two are linked by the identitylk(x, $p) = v(g(x, p))$ for utility maximizing demands $g(x, p))$. Substituting (10) into (9) yields