for parameters y and)3, the first-order conditions of which are readily solved to give the demand functions

$$p_i q, = piy, \pm)3,(x - p \bullet -y). \tag{6}$$

In practice, the first-order conditions are rarely analytically soluble even for quite simple formulations (e.g. Houthakker's (1960) "direct addilog" $u = Ece_i e.)$, nor is it at all straightforward to pass back from given demand functions to a closed form expression for the utility function underlying them, should it indeed exist.

The generic properties of demands are frequently derived from (3) by total differentiation and matrix inversion to express dq as a function of dx and dp, the so-called "fundamental matrix equation" of consumer demand analysis, see Barten (1966) originally and its frequent later exposition by Theil, e.g. (1975b, pp. 14f1), also Phlips (1974, 1983, p. 47), Brown and Deaton (1972, pp. 1160-2). However, such an analysis requires that *u(q)* be twice-differentiable, and it is usually assumed in addition that utility has been monotonically transformed so that the Hessian is non-singular and negative definite. Neither of these last assumptions follows in any natural way from reasonable axioms; note in particular that is is *not* always possible to transform a quasi-concave function by means of a monotone increasing function into a concave one, see Kannai (1977), Afriat (1980). Hence, the methodology of working through first-order conditions involves an expansive and complex web of restrictive and unnatural assumptions, many of which preclude consideration of phenomena requiring analysis. Even in the hands of experts, e.g. the survey by Barten and Bohm (1980), the analytical apparatus becomes very complex. At the same time, the difficulty of solving the conditions in general prevents a close connection between preferences and demand, between the a priori and the empirical.

1.3. Duality, cost functions and demands

There are many different ways of representing preferences and great convenience can be obtained by picking that which is most appropriate for the problem at hand. For the purposes of generating empirically useable models in which quantities are a function of prices and total expenditure, dual representations are typically most convenient. In this context, duality refers to a switch of variables, from quantities to prices, and to the respecification of preferences in terms of the latter. Define the *cost function,* sometimes *expenditure function,* by

$$c(u, p) = (min p \bullet q; u(q) \tag{7}$$