ued demand functions. Empirically, flats are important because they represent perfect substitutes; for example, between *S* and *T* on *B,* the precise combination of $q_1$ and $q_2$ makes no difference and this situation is likely to be relevant, say, for two varieties of the same good. Non-differentiabilities occur at the kink points on the curves *B* and C. With a linear budget constraint, kinks imply that for relative prices within a certain range, two or more goods are bought in fixed proportions. Once again, this may be practically important and fixed relationships between complementary goods are often a convenient and sensible modelling strategy. The n-dimensional analogue of the utility function corresponding to C is the fixed coefficient or Leontief utility function

$$v(q) = \min\{ a_1q_1, a_2q_2, \quad a_nq_n \} . \tag{2}$$

For positive parameters $a_n$. Finally curve *A illustrates* the situation where $q_2$ is essential but $q_1$ is not. As $q_2$ tends to zero, its marginal value relative to that of $q_1$ tends to infinity along any given indifference curve. Many commonly used utility functions impose this condition which implies that $q_2$ is always purchased in positive amounts. But for many goods, the behavior with respect to $q_1$ is a better guide; if $p_1 > p_2 8$, the consumer on indifference curve *A* buys none of $q_1$. Data on individual households always show that, even for quite broad commodity groups, many households do not buy all goods. It is therefore necessary'to have models that can deal with this fact

*1.2. Lagrangians and matrix methods*

If *v(q)* is strictly quasi-concave and differentiable, the maximization of utility subject to the budget constraint can be handled by Lagrangian techniques. Writing the constraint $p \cdot q = x$ for price vector $p$ and *total expenditure x,* the first-order conditions are

*cu(q)* _lip (3)

which, under the given assumptions, solve for the demand functions

$$q_i = g_i(x, p). \tag{4}$$

For example, the *linear expenditure system* has utility function

$u=$ *(5)*