

Suppose that the parameter of interest is β , the population-weighted average of β_1 and β_2 ; given the two coefficients, this could be obtained by weighting each by the inflation factor for its sector. For example, if w_i is the marginal propensity to consume in each sector, the population-weighted average β would be the marginal propensity to consume out of a randomly allocated unit of currency, a quantity that is often of interest in discussions of tax and benefit reform.

As with the case of estimating the population mean, it is immediately clear that the (unweighted) OLS estimator using all of the data is biased and inconsistent. Instead, we might follow the principle of the previous subsection, weighting each household by the number of households that it represents in the survey, and compute the weighted estimator

$$(\text{E } w_i y_i) / (\text{E } w_i x_i) \quad (10)$$

where w_i is the normalized inflation factor. This estimator converges, not to β , but to

$$\text{plim } \frac{w_2 \sigma_1^2 (\beta_1 - \beta_2) + \beta_2 (\sigma_1^2 + \sigma_2^2)}{w_2 \sigma_1^2 + \sigma_1^2 + \sigma_2^2} \quad (11)$$

where σ_1^2 and σ_2^2 are the (population) variances of x in each of the two sectors. Unlike the unweighted estimator, this quantity at least has the (limited) virtue of being independent of sample design; indeed, as is to be expected from the general argument for inflation factors, it is what OLS would give if applied to the data from the whole population (see Dumouchel and Duncan (1983)). However, it is not equal to the parameter of interest β unless either $w_1 = w_2$, or $\sigma_1^2 = \sigma_2^2$; the former is ruled out by hypothesis, and there is no reason to suppose that the latter will hold in general.

Of course, the fundamental issue here is not the sample design but the fact that the regression is not homogeneous within the population being studied. As such, the problem is not a sampling issue exactly the same issues arise in regressions using pooled time-series for a cross-section of countries but a heterogeneity issue, and it comes to the fore in the sampling context because it is heterogeneity that justifies the stratification in the first place. As a result, it is often plausible that behavioral parameters will differ across strata, just as they are likely to vary across countries. When this is not the case, and regression coefficients are identical, then both weighted and unweighted regressions are unbiased and consistent, and the Gauss-Markov theorem tells us that the *unweighted* regression is to be preferred. If instead the regression coefficients differ by strata, that fact has to be explicitly faced and cannot be finessed by running regressions weighted by inflation factors. Such recommendations were