Solving …

1. $$\frac{a-\iota(p_t)}{p_t}+g(p_t)+\mu_t+\sigma\sigma_t^p-r=-\frac{f'(\eta_t)}{f(\eta_t)}\sigma_t^\eta(\sigma+\sigma_t^p)$$

where

$$\mu_t^p=\frac{p'(\eta_t)[(r-g(p_t)+\sigma^2)(\eta_t-p_t)+a-\iota(p_t)]+\frac{1}{2}(\sigma_t^\eta)^2p''(\eta_t)}{p_t(1-p'(\eta_t))}\quad\Rightarrow$$

$$\sigma_t^p=\frac{p'(\eta_t)\sigma(p_t-\eta_t)}{p_t(1-p'(\eta_t))}$$

$$\sigma_t^\eta=\frac{\sigma(p_t-\eta_t)}{1-p'(\eta_t)}$$

2. $$(\rho-r)f(\eta)=f'(\eta)((r-g(p_t)+\sigma^2)(\eta-p_t)+a-\iota(p_t)+p_t\mu_t^p)+\frac{1}{2}(\sigma_t^\eta)^2f''(\eta_t)$$

from $(\rho\text{-}r)\, f(\eta) = \mu_t^f$

- 4 boundary conditions: $p(0) = \underline{p}$, $p'(\eta^*) = 0$, $f(\eta^*) = 1$, $f'(\eta^*) = 0$
- Solve for $p(\eta)$, $p'(\eta)$, $p''(\eta)$, $f(\eta)$, $f'(\eta)$, $f''(\eta)$