

# Solving ...

- Bellman equation:  $\rho f(\eta_t) n_t dt = \max_k E[d(f(\eta_t) n_t)]$  (when  $f(\eta) > 1$ )  
 $E[d(f(\eta_t) n_t)] = \mu_t^f n_t dt + f'(\eta_t) \sigma_t^n k_t p_t (\sigma + \sigma_t^p) dt + f(\eta_t) (r n_t + (a - \iota(p_t)) k_t + k_t p_t (g(p_t) - r + \mu_t^p + \sigma \sigma_t^p))$
- FOC: 
$$\frac{(a - \iota(p_t))/p_t + g + \mu_t^p + \sigma \sigma_t^p - r}{\text{expected excess return on capital}} = \frac{-f'(\eta_t)/f(\eta_t) \sigma_t^n (\sigma + \sigma_t^p)}{\text{risk premium from precautionary motive}}$$
- Using FOC, Bellman equation simplifies to
  - $(\rho - r) f(\eta_t) = \mu_t^f$
- Derive  $\mu_t^p, \mu_t^f, \sigma_t^n, \sigma_t^p$  in terms of  $p', p'', f', f''$  to obtain ODE for  $p(\eta)$  and  $f(\eta)$