

Equilibrium

- Aggregate variables
- State variable

$$N_t, K_t$$

$$\eta_t = N_t/K_t$$

1. Internal investment

- Entrepreneur takes price p_t as given

$$\max_{i_t} p_t k_t (\Phi(i_t/k_t) - \delta) - i_t$$

$$\text{FOC: } p_t \Phi'(i_t/k_t) - 1 = 0 \quad (\text{Tobin's } q)$$

$$\Rightarrow \iota(p_t) = i_t/k_t, \text{ rate of investment per unit of capital}$$

- $g(p_t) := \Phi(i_t/k_t) - \delta =$ (optimized) growth rate of capital
- Note : $g(p_t) = -\infty$ for $p_t < (a - i^*)/(r - g)$: capital is sold to unproductive HH

2. External investment

- Given price process
- Solvency constraint

$$k_t$$

$$dp_t/p_t = \mu_t^p dt + \sigma_t^p dZ_t$$

$$n_t \geq 0$$

3. When to consume?

$$dc_t$$

4. Market clearing:

$$\text{Total demand} = K_t$$

dynamic
optimization
+ market
clearing