Bad loans have a higher probability of default $\varepsilon + k$, where $k > 0$ is a constant. The c.d.f. of the realised value of one dollar face value of bad loans is denoted by $F_B(\cdot)$. We assume that F_G cuts F_B precisely once from below. In other words, the bad loans have a lower expected payoff, but they have greater dispersion of outcomes.

A bank is both a lender and a borrower. Denote by φ the bank's notional debt per one dollar face value of its loan portfolio. The bank defaults if the realised value of one dollar's face value of loans falls below φ. Therefore, φ is the strike price of the embedded put option arising from limited liability. The bank faces a leverage cap through an upper limit on φ.

Denote by $\pi_G(\varphi)$ the option value of default given notional debt φ under the good portfolio, and denote by $\pi_B(\varphi)$ the option value under the bad portfolio. The incentive compatibility constraint for the bank to choose the good portfolio is

$$E(w_G) - [\varphi - \pi_G(\varphi)] \geq E(w_B) - [\varphi - \pi_B(\varphi)] \tag{10}$$

where $E(w_G)$ is the expected realised value of one dollar's face value of good loans and $E(w_B)$ is the analogous expected realised value for bad loans. Writing $\Delta\pi(\varphi) = \pi_B(\varphi) - \pi_G(\varphi)$, (10) can be written more simply as

$$\Delta\pi(\varphi) \leq k \tag{11}$$

The left hand side is the additional option value to default from the bad portfolio and the right hand side is the difference k in expected realised values between the good and bad portfolios. Incentive compatibility entails keeping leverage low enough that the higher option value to default does not exceed the greater expected payoff of the good portfolio. Our solution rests on being able to solve for a unique leverage level given by the following lemma.

Lemma 1 *There is a unique φ that solves $\Delta\pi(\varphi) = k$.*

We prove Lemma 1. From risk neutrality, $\pi_G(\varphi)$ is the expected payoff of the put option on one dollar face value of loans with strike price φ

$$\begin{aligned} \pi_G(\varphi) &= \int_0^{\varphi} (\varphi - s) f_G(s)\, ds \\ &= \varphi F_G(\varphi) - \int_0^{\varphi} s f_G(s)\, ds \qquad (12) \\ &= \int_0^{\varphi} F_G(s)\, ds \qquad (13) \end{aligned}$$