



Figure 6. The two charts plot the densities over realised values of a loan portfolio with face value of 1 dollar. The left hand charts plots densities when $\rho = 0.1$ and ϵ is varied from 0.1 to 0.3. The right hand chart plots densities when $\epsilon = 0.2$ and ρ varies from 0.01 to 0.3.

where ϵ is the probability of default of borrower j , defined as $\epsilon = \Phi(-d_j)$ and Φ is the standard normal c.d.f.

Conditional on Y , defaults are independent. In the limit where the number of borrowers becomes large the realised value of the loan portfolio with face value of 1 dollar can be written as a deterministic function of Y , by the law of large numbers. Defaulted loans have zero recovery value. The realised value of one dollar face value of loans is the random variable $w(Y)$ defined as:

$$\begin{aligned}
 w(Y) &= \Pr\left(\sqrt{\rho}Y + \sqrt{1-\rho}X_j \geq \Phi^{-1}(\epsilon) \mid Y\right) \\
 &= \Phi\left(\frac{Y\sqrt{\rho} - \Phi^{-1}(\epsilon)}{\sqrt{1-\rho}}\right)
 \end{aligned} \tag{8}$$

The c.d.f. of w is then given by

$$\begin{aligned}
 \Pr(w \leq z) &= \Pr(Y \leq w^{-1}(z)) \\
 &= \Phi(w^{-1}(z)) \\
 &= \Phi\left(\frac{\Phi^{-1}(\epsilon) + \sqrt{1-\rho}\Phi^{-1}(z)}{\sqrt{\rho}}\right)
 \end{aligned} \tag{9}$$