THE CEMRACS 2000 TEST CASE FOR NUCLEAR WASTE DISPOSAL

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Abstract

The purpose of this test case is to motivate the community of scientific computing to contribute to the understanding and control of the "medium and far field" problem which arises in nuclear waste management. From the mathematical point of view the problem is of convection diffusion type but the parameters vary very much from one layer of ground material to another.

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1 Introduction

Nuclear waste from nuclear electricity is a big problem because it is dangerous to human health for 10 million years.

Whether or not nuclear power plants are discontinued there is already in Europe a large amount of dangerous radio-active material which cannot be entirely recycled and which has to be disposed of. At present it is stored in secure places in containers. But it is likely that nuclear engineering will be forgotten and so it is not safe to keep these containers on the earth surface for long. Among the possible solutions one is to build an underground repository and make sure that it does not leak to the surface for 10 millions of years. Numerical simulation is an important tool to evaluate the security of such a repository.

The repository is dug at a depth of 260m (meters) in a clay layer which has above it a layer of lime stone and a layer of marl and below it layer of dogger lime stone. Water flows slowly through these porous media and convects the radioactive materials once the containers have broken; there is also a dillution effect which in mathematical terms is a diffusion. The problem is difficult for two reasons:

1. The radioactive elements leak from the containers into the clay over a period of 200 years which is small compared with the millions of years over which convection and diffusion are active.

2. The convection and diffusion constants are widely different from one layer to another; for instance, in the clay there is almost no diffusion while in the others diffusion and convection are equally important.

2 The Geometry

2.1 Case 1

The computational domain is in a rectangle $\mathcal{O} = (0, 2500) \times (0, 695)$ in meters. The horizontal layers of clay lime stone and marl are

- dogger $0 < y < 200$

	Dogger	Clay	lime stone	marl
K(m/an^2)	3.1536e-5	6.3072	3.1536e-6	25.2288

Table 1: Permeability tensor in the four layers

- clay $200 < y < 295$
- lime stone $295 < y < 595$
- marl $595 < y < 695$.

The repository is a rectangle in the clay layer

$$R = \{(x, y) \in (1800, 2100) \times (250, 270)\}$$

Thus the computational domain is $\Omega = \mathcal{O} \backslash R$.
For this domain the computation should be carried for $t \in (0, T)$ with $T = 100.000$ years.

2.2 Case 2

Let $\mathcal{O}_L = (-22500, 2500) \times (0, 695)$. The computational domain is $\Omega = \mathcal{O}_L \backslash R$.
For this domain the computation should be carried for $t \in (0, T)$ with $T_L = 10.000.000$ years.

3 The Flow

It is assumed that all layers are saturated with water and that boundary loads are stationary so that the flow is independent of time. Darcy's law gives the velocity $\mathbf{u}$ in terms of the hydro-dynamic load $H = P/g + y$:

$$\mathbf{u} = -K \nabla H$$

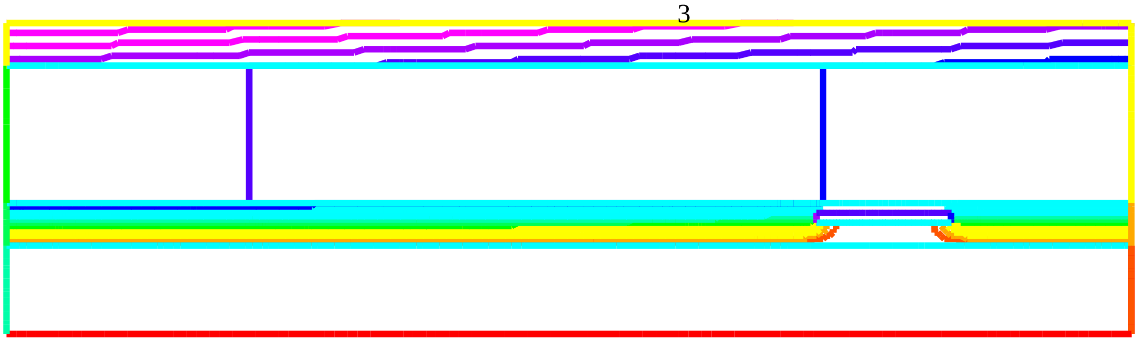
where K is the permeability tensor (given in Table 3), P the pressure and g Newtown's constant. Conservation of mass ($\nabla \cdot (\rho u) = 0$, where ρ is the density) implies that

$$\nabla \cdot (K \nabla H) = 0 \quad \text{in } \Omega \tag{1}$$

because water is assumed incompressible (ρ is constant). Boundary conditions are

$$\begin{aligned}
&H = 286.3 \text{ on } \{2500\} \times \{0, 200\} \\
&H = 211 \text{ on } \{2500\} \times \{295, 595\} \\
&H = 180 + 16 * x/2500 \text{ on } \{0, 2500\} \times \{0\} \\
&H = 200 \text{ on } \{0\} \times \{0, 200\} \\
&H = 286 \text{ on } \{0\} \times \{295, 595\} \\
&\frac{\partial H}{\partial n} = 0 \text{ elsewhere.}
\end{aligned}$$

Level lines of the hydrodynamic load



		I^{129}			Pu^{242}	
	De(m^2/an)	α_L(m)	α_T(m)	De(m^2/an)	α_L(m)	α_T(m)
Dogger	5.0e-4	50	1	5.0e-4	50	1
Clay	9.48e-7	0	0	4.42e-4	0	0
lime stone	5.0e-4	50	1	5.0e-4	50	1
marl	5.0e-4	0	0	5.0e-4	50	0

Table 2: Diffusion coefficients for the radioactive elements in the 4 layers

4 The Radioactive Elements

There are two species of particular interest, iodine 129 and Plutonium 242. Both escape from the repository cave into the water and their concentrations $C_i, i = 1,2$ is given by two independent convection-diffusion equations:

$$R_i\omega_i(\frac{\partial C_i}{\partial t} + \lambda_i C_i) - \nabla \cdot (D_i \nabla C_i) + \mathbf{u} \cdot \nabla C_i = 0 \quad \text{in } \Omega \times (0,T) \quad i = 1,2. \quad (2)$$

where

- R_i is the latency, 1 for I^{129}, 10^5 for Pu^{242} in the clay and 1 elsewhere for both.
- ω, the porosity, is 0.001 for I^{129}, 0.2 for Pu^{242} in clay and 0.1 elsewhere for both,
- $\lambda_i = \log 2/T_i$ and T_i is the half life of the element : $1.57.10^7$ for I^{129}, 3.7610^5 for Pu^{242} (in years).
- D_i, the diffusion coefficients are

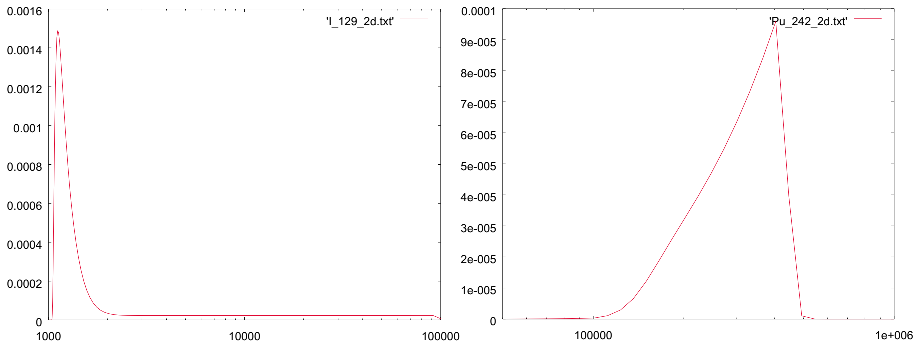
  $$D_i = \begin{pmatrix} D_{Li} & 0 \\ 0 & D_{Ti} \end{pmatrix} \text{ with } D_{Mi} = De_i + \alpha_M |\mathbf{u}|, \;\; M = L, T, \; i = 1,2.$$

  and the coefficients are given in the Table 4.

4.1 Initial and Boundary Conditions

At time zero the containers begin to leak and the radioactive elements spread. So C_i are zero at time zero and zero on the boundary of the domain where $\mathbf{u} \cdot n < 0$, where n is the outer normal. If $\mathbf{u} \cdot n \geq 0$ then the radio-nucleides escape and so $\frac{\partial C_i}{\partial n} = 0$.

There is a constant in space varying in time Dirichlet boundary condition on R, the boundary of the repository. It is a function of time, given by figures 2 and 3.



Figures 2 & 3: Release of Iodine and Plutonium as a function of time

For the first one we could use an analytic representation like

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function source(t) =0.0015*min(t,65)*exp(-max(t-150,0)/100)/65;
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5 The Test Cases

5.1 Test Case I

Compute C_1 by solving (1)(2) in $(\mathcal{O}\backslash R) \times (0, T)$

5.2 Test Case II

Compute C_1 and C_2 by solving (1)(2) in $(\mathcal{O}_L\backslash R) \times (0, T_L)$

5.3 Presentation of Results

Level lines of C_i at various instant of times. In order to compare the results a zip disk with the data in text format must be provided with the following information:
- a file for every 20 years up to year 300
- a file very 100 years from year 400 to the end

Each file must have
`x,y,Ci(x,y)` at every Finite Difference node
or
`x,y, Ci(x,y)` at every Finite Element or Finite Volume node and one extra file for the connectivity of the nodes.

5.4 The Difficulty

Beside the fact that the source terms are active for a small period of time, we show in Table 5.4 below the average values of the diffusion coefficients and of the convection

	Dogger	Clay	lime stone	marl
lambda1R1w	4.41497e-09	4.41494e-11	4.41498e-09	4.41495e-09
R1w	0.1	0.001	0.1	0.1
DL	57.5284	9.47996e-07	4.15073	0.0005
DT	1.15106	9.47996e-07	0.0835049	0.0005
u	-0.00434425	-0.00114385	-0.0274287	-0.00268802
v	1.14722	1.22948	0.0577332	0.102462

Table 3: Average values of the convection and diffusion coefficients in the 4 layers for test case I

speed in each layer for case I.