Combination Forecasting for Greek GDP Using Multi-Step Cross-Validation

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Abstract

This paper develops point and interval forecasts of cumulative Greek GDP growth for the period 2015Q2-2017Q1. The forecasts are based on combining 1701 separate regression forecasts based on a broad set of leading indicators. The forecast combination weights are selected by minimizing the multi-step leave-h-out cross-validation criteria, which is an estimate of the multi-step mean-squared forecast error. We forecast that Greek GDP will decline by 2% over the next several quarters, and not achieving positive growth until late 2016. These estimates are calculated under the strong assumption of no regime shift, and thus assume that a finance deal is reached with the international creditors, that the banking system is re-opened without capital controls, and that Greece stays in the Euro zone.

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1 Introduction

The Greek debt crisis is at the center of the 2015 financial news. Much of the technical discussion concerns the sustainability of the debt payments and the appropriate targets for the primary surplus (the difference between receipts and expenditures excluding interest and principle payments) as a percentage of GDP. This discussoin is layered on top of the fact that the Greek economy is in a multi-year depression. This paper attempts to address one narrow issue: What is our best estimate of the future path of Greek GDP over the next two years?

Answering this question is fraught with pitfalls, since the country is in the midst of a financial crisis and possible exit from the Euro zone. Our forecasts cannot hope to deal with these issues and thus are not directly addressed here. Instead, our forecasts will be constructed assuming that there is no regime shift; that the economic relationships will continue along a similar path as in the path. Thus these estimates are predicated on the assumption that a finance deal is reached with the its international creditors, that the banking system is re-opened without capital controls, and that Greece stays in the Euro zone.

Our forecasts will use the multi-step leave-h-out cross-validation combination methods of Hansen (2010) and Cheng and Hansen (2015), which builds on the combination methods of Hansen (2007), Hansen (2008) and Hansen and Racine (2012). The forecast combination weights are selected by minimizing a cross-validation criteria, which is an estimate of the multi-step mean-squared forecast error (MSFE).

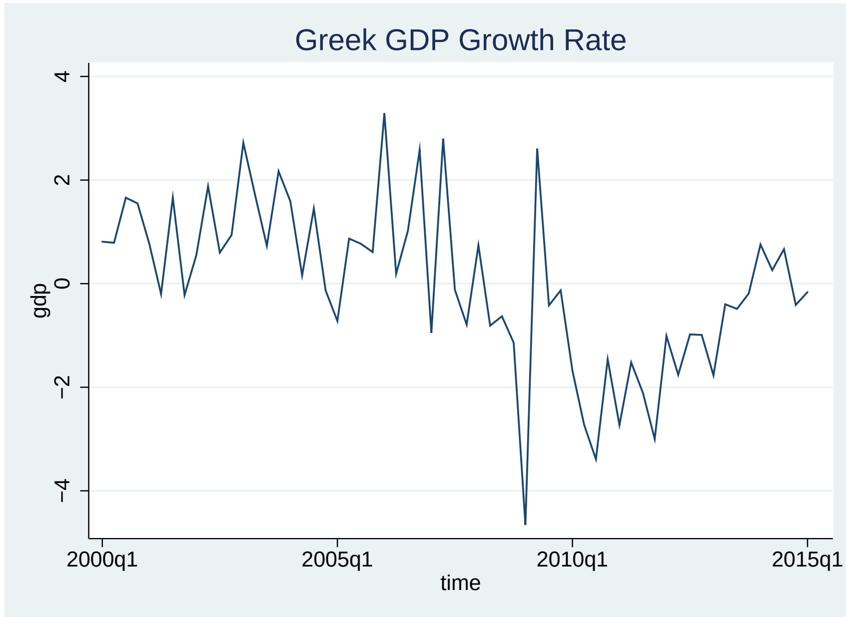
Other recent papers which build forecasting models for Greek GDP include Monokroussos and Thomakas (2012), Kourentzesa and Petropoulosa (2014) and Lamprou (2015).

Those interested in the numerical forecast but not the methodology can skip directly to Section 7.

2 Data

Historical Greek GDP growth for 2000-2015 is plotted in Figure 1. We use the series as reported by the Greek statistical agency, and is quarterly percentage change (not annualized).

The series appears uneventful until the sharp drop (-4.66) in 2009Q1. GDP growth was negative for every quarter from 2009Q3-2013Q3, and then slipped negative again for the past two quarters.



Greek GDP Quarterly Growth Rate 2000-1015

Our goal is to forecast the next eight observations for GDP cumulative growth, which are the periods 2015Q2-2017Q1.

The predictors we consider are

Table 1: Predictors

Building Permits	BP
Unemployment Rate	UR
Industrial Production Growth Rate	IP
Retail Sales Growth Rate	RG
Retail Turnover Growth Rate	RT
Athens Stock Index Return	SR
Economic Sentiment Indicator	ESI
Spread=Yield on Greek 10-Year Bond - Yield on German 10-Year Bond	SP

To simplify the analysis, monthly variables are aggregate to quarterly to match the GDP observation frequency. All variables are available for 2000Q1-2015Q1, and for the last three until 2015Q2.

The industrial production, retail sales and retail turnover series were all converted to growth rates to match the GDP series and since the variables appear to be non-stationary. The building permits, unemployment rate, and confidence index were retained as levels variables. The economic sentiment indicator is a survey measure by the IOBE and its successful role in forecasting Greek GDP growth has been demonstrated by Monokroussos & Thomakas (2012).

3 h-step Direct Linear Forecasting

By an "h-step" forecast we mean forecasting a variable y_{t+h} given information in period t. For quarterly data, the horizon h is measured in quarters. We are interested in forecasting cumulative growth. We thus define the h-step dependent variable y_{t+h} as the cumulative percentage growth from t to $t+1$. Thus y_{n+4} is the total growth over the next year and y_{n+8} is the total growth over the next two yeras.

A linear h-step direct forecasting model for y_{t+h} given a set of regressors $\boldsymbol{x}_t' = (x_{1i}, ...x_{kt})$ takes the form

$$\begin{aligned} y_{t+h} &= a_0 \sum_{i=1}^{k} a_i x_{it} + e_{t+h} \\ &= a_0 + \boldsymbol{x}_t' \boldsymbol{a} + e_{t+h} \\ \mathbb{E}\left(e_{t+h} \mid \boldsymbol{x}_t\right) &= 0 \end{aligned} \tag{1}$$

The variables $\boldsymbol{x}_t$ are known at time t but are not necessarily dated at time t. Thus $\boldsymbol{x}_t$ can include lagged dependent variables (e.g. y_t, y_{t-1}) lags of predictors, and even variables dated ahead of time t. In our example, three variables (stock returns, economic sentiment indicator, bond spread) are observed one quarter ahead of GDP growth, so we include include their time $t+1$ values in $\boldsymbol{x}_t$. Thus a 1-step forecast involves the regression of GDP growth on the contemporeneous values of these variables. This is valid from a forecasting point of view but should not be interpreted causally, and is based from the nowcasting literature. Including the 2015Q2 values of these variables in our forecasts means that to the extent that the financial crisis influenced the values of these variables in Q2, the current financial crisis is reflected in our forecasts.

The relationship between economic growth and bond spreads is well documented. During periods of expected economic downturns, bond default risk increases, the market price of bonds falls and the spread over less-risky bonds will increase. We specify the bond spread in terms of the difference between the yields on 10-year Greek and German bonds as these series are well established, both are denominated in Euro, and German bonds are perceived in the market as near riskless. The relationship between economic growth and bond spreads is also possibly a non-linear relationship. Hence we allow for flexibility in the regression by modeling it as a continuous linear spline with a single knot at 8%. The knot was selected informally by data inspection and some experimentation.

Least-squares estimates of the one-step-ahead ($h = 1$) model with no additional lags is presented in Table 2.

Table 2
One-Step Ahead Forecast Regression for Greek GDP Growth

	Coefficient	s.e.
GDP_{t-1}	-0.35	0.15
BP_{t-1}	0.80	0.29
UR_{t-1}	0.07	0.08
IP_{t-1}	-0.10	0.05
RG_{t-1}	0.01	0.08
RT_{t-1}	-0.09	0.10
SR_t	0.015	0.010
ESI_t	0.09	0.04
SP_t	-0.50	0.12
Spline_t	0.55	0.12
Intercept	0.95	4.3

The regression suggests that variables which are positively related with one-step GDP growth include building permits, stock returns, and the economic sentiment index. The bond spread is negatively related with growth up to spreads of 8%. For spreads above 8% the effect is effectively capped.

To allow for dynamic effects, we want to include two lags of each forecast variable in the regression. This would require $k = 20$ regressors. As there are only $61 - h$ observations to estimate each forecast regression, it is effectively impossible to estimate the full regression model. Another way of viewing it is that the estimation variance would be extremely high. In the next sections we explore using forecast combination methods to reduce the estimation variance while controlling omitted variable bias.

4 Sub-Models

A sub-model of (1) is a subset of the regressors. Specifically, let $m = 1, ..., M$ denote a set of models, where for each m we denote $\boldsymbol{x}_t(m)$ as a $k(m)$ subset of the variables in $\boldsymbol{x}_t$. The maximum number of sub-models is 2^k, but not all sub-models need be considered. In our application we will restrict the models so that lags are only included sequentially and the spline term is only included sequentially. Thus we will consider models which include $\{GDP_{t-1}\}$ and $\{GDP_{t-1}, GDP_{t-2}\}$ but not those which include $\{GDP_{t-2}\}$ only.

For each regressor subset we can define an approximating regression function by standard linear projection, written as

$$\begin{aligned} y_{t+h} &= \boldsymbol{x}_t(m)'\boldsymbol{a}(m) + e_{t+h}(m) \qquad (2) \\ \sigma^2(m) &= \mathbb{E}\left(e_{t+h}(m)^2\right) \end{aligned}$$

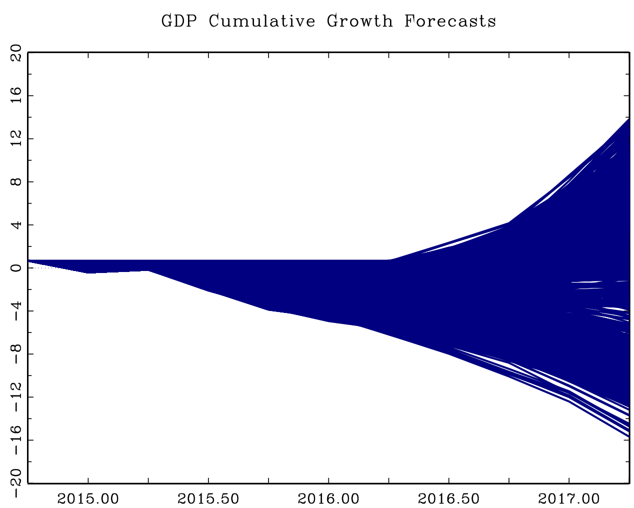
In equation (2) the equation error $e_{t+h}(m)$ is a projection error not necessarily a MDS as it generally contains omitted variables.

For each model m, the coefficients and residuals may be estimated by least-squares, and out-of-sample point forecasts calculated in the usual way.

$$\begin{aligned}\widehat{\boldsymbol{a}}(m) &= \left(\mathbf{X}(m)'\mathbf{X}(m)\right)^{-1}\mathbf{X}(m)'\mathbf{y}\\ e_{t+h}(m) &= y_{t+h} - \boldsymbol{x}_t(m)'\widehat{\boldsymbol{a}}(m)\\ \widehat{y}_{n+h}(m) &= \boldsymbol{x}_n(m)'\widehat{\boldsymbol{a}}(m)\end{aligned}$$

As we mentioned before, when the number of regressors k is large relative to the sample size n, it generally does not make sense to forecast based on the full model as will be over-parameterized. However, using an arbitrarily selected sub-model is fraught with danger as the choice generally makes a great deal of difference. To illustrate, in Figure 2 we display plots of the time-paths of the point forecasts from 1701 submodels. The point forecasts are highly divergent. Hence selecting one forecast arbitrarily from this set is highly dangerous.

GDP Cumulative Forecasts From 1701 Submodels



5 Forecast Combination

Given a set $m = 1, ..., M$ of models with associated point forecasts $\widehat{y}_{n+h}(m)$, a forecast combination is the weighted average

$$\widehat{y}_{n+h}(\mathbf{w}) = \sum_{m=1}^{M} w(m)\widehat{y}_{n+h}(m). \tag{3}$$

where $w(m)$ are forecast weights and we write $\mathbf{w} = (w_1, ..., w_M)$. We constrain the weights to be non-negative and sum to 1. Thus $\mathbf{w}$ is an element of the M-dimensional unit simplex $\mathcal{H}$.

A forecast combination partially circumvents the arbitrariness of conditioning on a single model as it instead takes an ensemble average.

Forecast combination requires the selection of the weight vector $\mathbf{w}$. There is a substantial literature on selection of the weight vector in forecasting, but much of the literature is heuristic and non-rigorous.

A method based on a direct estimate of the MSFE is presented in Hansen (2010) and Cheng and Hansen (2015). These papers show that a direct and approximately unbiased estimate of the MSFE of the forecast combination $\widehat{y}_{n+h}(\mathbf{w})$ is provided by the leave-h-out cross-validation criteria (discussed below). This motivates selecting the weight vector by minimization of the cross-validation criteria. This is a natural extension of traditional model selection methods to model averaging.

The leave-h-out cross-validation criteria is calculated as follows. First, for model m, horizon h, and observation t the leave-h-out estimates are

$$\widetilde{\boldsymbol{a}}_{t,h}(m) = \left(\sum_{|j-t| \geq h} \boldsymbol{x}_j(m) \boldsymbol{x}_j(m)' \right)^{-1} \left(\sum_{|j-t| \geq h} \boldsymbol{x}_j(m) y_{j+h} \right).$$

This is the least-squares estimates of the sub-model (2) using all observations except those within $h-1$ periods of t. For one-step forecasts ($h=1$) this excludes observation t, for two-step ($h=2$) this excludes $(t-1, t\,, t+1)$, etc. The associated leave-h-out residual is

$$\widetilde{\boldsymbol{e}}_{t+h}(m) = y_{t+h} - \widetilde{\boldsymbol{a}}_{t,h}(m)' \boldsymbol{x}_t(m).$$

This is the forecast error which occurs when you forecast y_{t+h} using observations excluding those near time t.

For any set of weights $\mathbf{w}$, the combination leave-h-out residual is

$$\begin{aligned} \widetilde{\boldsymbol{e}}_{t+h}(\mathbf{w}) &= y_{t+h} - \sum_{m=1}^{M} w(m) \widetilde{\boldsymbol{a}}_{t,h}(m)' \boldsymbol{x}_t(m) \\ &= \sum_{m=1}^{M} w(m) \widetilde{\boldsymbol{e}}_{t+h}(m) \end{aligned}$$

The leave-h-out cross-validation criteria is

$$\begin{aligned} CV_h(\mathbf{w}) &= \frac{1}{n} \sum_{t=1}^{n} \widetilde{\boldsymbol{e}}_{t+h}(\mathbf{w})^2 \\ &= \frac{1}{n} \sum_{t=1}^{n} \left(\sum_{m=1}^{M} w(m) \widetilde{e}_{t+h}(m) \right)^2 \\ &= \mathbf{w}' \widetilde{\mathbf{S}} \mathbf{w} \end{aligned} \tag{4}$$

where $\widetilde{\boldsymbol{S}} = \frac{1}{n}\tilde{\boldsymbol{e}}'\tilde{\boldsymbol{e}}$ and $\tilde{\boldsymbol{e}}$ is the $n \times M$ matrix of stacked leave-h-out prediction errors.

By definition, CV-selected weight vector is the choice which minimizes the CV criteria over the unit simplex:

$$\widetilde{\mathbf{w}}_h = \underset{\mathbf{w}\in\mathcal{H}}{\operatorname{argmin}}\, CV_h(\mathbf{w}). \tag{5}$$

Given this choice, the combination point forecast is (3) evaluated using the weights $\widetilde{\mathbf{w}}_h$:

$$\widehat{y}_{n+h} = \widehat{y}_{n+h}(\widetilde{\mathbf{w}}_h) = \sum_{m=1}^{M} \widetilde{w}_h(m)\widehat{y}_{n+h}(m).$$

Equation (4) shows that the CV criteria is a simple quadratic function of the weight vector $\mathbf{w}$. Numerical minimization is non-standard due to the need to impose non-negativity constraints. (It is not clear if *non-negativity* per se is necessary, it may be sufficient to bound the weights above -1, but constraints are certainly required to regularize the problem.) Quadratic minimization subject to inequality constraints is a quadratic programming problem for which numerical algorithms are widely available and computationally efficient. Since the constraint set (the unit simplex) has sharp edes the solution (5) is typically an edge solution, meaning that most models numerically receive 0 weight. As a practical matter, when using cross-validation to select the combination weights among a very large set of sub-models, the solution will put positive weight on a handful of sub-models.

6 Selected Models and Weights for Greek GDP

The number of models $2^{20} = 1,048,600$ is too large for feasible implementation, so some simplifications were made. First, preliminary analysis found that 3 variables (industrial production, retail sales volumn, retail sales turnover) did not appear with positive weights at any horizon, so they were omitted from the analysis. Second, we restricted the models so that lags are only included sequentially and the spline term is only included sequentially, as discussed early. With these restrictions we have 1,701 models which is feasible for numerical implementation.

Table 3 below shows the sub-models selected for the one-step forecast by minimization of $CV_1(\mathbf{w})$. Six sub-models received positive weight (the remaining 1,695 receive 0 weight). The check marks indicate which regressors were included in each model, listed in order from highest weight to lowest weight. The last row in the Table shows the point forecast from each sub-model. The latter range from -0.5 to -1.7. The weighted average is $\widehat{y}_{2015Q2} = -1.13$ which is our point forecast for 2015Q2.

Table 3:
One-Step Forecast Combination Sub-Models and Weights

GDP_{t-1}	✓	✓			✓	✓
GDP_{t-2}	✓				✓	
BP_{t-1}		✓		✓		
BP_{t-2}		✓				
UR_{t-1}	✓	✓		✓	✓	✓
UR_{t-2}						✓
SR_t			✓			✓
ESI_t	✓		✓		✓	
ESI_{t-1}	✓				✓	
SP_t	✓	✓			✓	✓
$Spline_t$	✓	✓			✓	✓
SP_{t-1}		✓			✓	✓
$Spline_{t-1}$		✓				✓
weight	0.28	0.24	0.13	0.12	0.12	0.10
$\widehat{y}_{n+1}(m)$	−1.7	−0.8	−0.5	−0.7	−1.7	−0.8

In Table 4, 5, and 6 we present similar results for two-step, four-step, and eight-step forecasts. For the 8-step forecast the sub-model forecasts are quite diverse. 82% of the weight is placed on two models which give a point forecast of 4% growth, but the remaining 18% of the weight is placed on negative growth forecasts, including 5% weight placed on −8% growth. This shows the difficulty of forecasting at this horizon.

Table 4:
Two-Step Forecast Combination Sub-Models and Weights

GDP_{t-1}		✓		✓		
BP_{t-1}	✓		✓	✓	✓	
BP_{t-2}				✓	✓	
UR_{t-1}	✓	✓		✓	✓	
SR_t		✓				✓
SR_{t-1}		✓				
ESI_t		✓	✓			✓
ESI_{t-1}		✓	✓			✓
SP_t	✓	✓		✓		
$Spline_t$	✓	✓		✓		
SP_{t-1}	✓					
$Spline_{t-1}$	✓					
weight	0.38	0.30	0.11	0.07	0.07	0.05
$\widehat{y}_{n+2}(m)$	−1.2	−2.7	−3.1	−1.7	−1.3	−1.2

Table 5:
Four-Step Forecast Combination Sub-Models and Weights

GDP_{t-1}		✓	✓			✓			✓
BP_{t-1}	✓	✓	✓		✓			✓	✓
BP_{t-2}	✓	✓	✓					✓	✓
UR_{t-1}	✓	✓	✓	✓	✓	✓			✓
SR_{t}	✓								✓
SR_{t-1}	✓								✓
ESI_{t}			✓	✓		✓	✓	✓	
ESI_{t-1}			✓					✓	
SP_{t}	✓	✓		✓		✓	✓	✓	✓
Spline_{t}		✓		✓		✓	✓	✓	✓
SP_{t-1}								✓	✓
Spline_{t-1}								✓	✓
weight	0.27	0.23	0.11	0.10	0.08	0.07	0.06	0.04	0.03
$\widehat{y}_{n+4}(m)$	−0.8	−2.6	−1.1	−1.9	−2.4	−2.1	−3.6	−4.6	−2.0

Table 6:
Eight-Step Forecast Combination Sub-Models and Weights

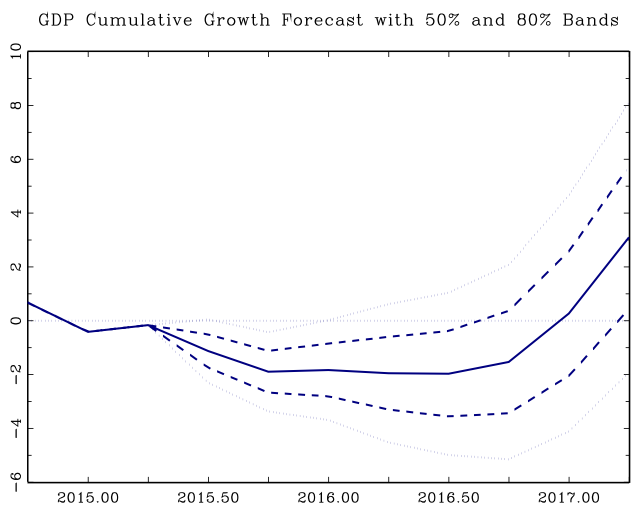
GDP_{t-1}			✓	✓	✓
GDP_{t-2}			✓	✓	✓
BP_{t-1}	✓	✓		✓	
BP_{t-2}	✓				
UR_{t-1}	✓	✓			
ESI_{t}	✓	✓		✓	
ESI_{t-1}	✓	✓		✓	
SP_{t}			✓	✓	
Spline_{t}				✓	
SP_{t-1}			✓	✓	
Spline_{t-1}				✓	
weight	0.45	0.37	0.05	0.05	0.03
$\widehat{y}_{n+8}(m)$	3.9	4.0	−2.6	−7.7	−2.0

7 Forecast Results

Our pont forecasts are displayed in Table 7 and in Figure 3 as a fan chart, along with 50% and 80% forecast intervals.

The point forecasts are that growth will be negative for the next two quarters (2015Q2 and 2015Q3), then 0% for the following three quarters, and positive for 2016Q4 and 2017Q1. The point forecasts are for the fall in GDP to reach a maximum of negative 2%, and for GDP to not grow beyond its current level until 2016Q4. The forecasted growth for the final two quarters is much stronger, so that the forecasted cumulative growth is positive 3% by 2017Q4. However, there is considerable uncertainty associated with this forecast.

The forecast intervals show that their is considerable uncertainty associated with the forecast. However, the intervals show that negative growth for the remainder of 2015 is highly likely.



GDP Cumulative Growth Forecasts with 50% and 80% Forecast Intervals

Table 7:
Cumulative Greek GDP Growth for 2015Q2 through 2017Q1

Horizon	Point Forecast	50% Interval	80% Interval
2015Q2	−1.1	(−1.7, −0.5)	(−2.3, 0.1)
2015Q3	−1.9	(−2.7, −1.1)	(−3.4, −0.4)
2015Q4	−1.8	(−2.8, −0.8)	(−3.7, 0.0)
2016Q1	−1.9	(−3.3, −0.6)	(−4.5, 0.6)
2016Q2	−2.0	(−3.6, −0.4)	(−5.0, 1.0)
2016Q3	−1.5	(−3.4, −0.4)	(−5.1, 2.1)
2016Q4	0.3	(−2.0, 2.6)	(−4.1, 4.7)
2017Q1	3.1	(0.5, 5.7)	(−1.9, 8.1)

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