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Design, Numerical Simulation of Jerk Circuit and Its Circuit Implementation

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Abstract

In this paper, in order to show some interesting phenomena of three dimensional autonomous ordinary differential equations, the chaotic behavior as a function of a variable control parameter, has been studied. The complex dynamical behaviors of the system are further investigated by means of eigenvalue structures and various attractors. The chaotic system examined in MATLAB 2010. The Oscillator circuit of the chaotic system is afterwards designed by using MultiSIM software and a typical chaotic attractor is experimentally demonstrated.

Keywords: Jerk circuit, chaotic, eigenvalue structure

1. Introduction

Chaos is generally defined as a state that exists between definite and random states. Chaos is very interesting nonlinear phenomenon and has been intensively studied during the last four decades. Chaos theory, as a branch of the theory of nonlinear dynamical systems, has brought to our attention a somewhat surprising fact: low-dimensional dynamical systems are capable of complex and unpredictable behavior. Chaos has been widely applied to many scientific disciplines: physics [1], biology [2]-[3], economics [4], engineering [5]-[7], finance [8], chemical [9], population dynamics [10]-[11], psychology [12], image encryption [13]-[14], text encryption [15] and robotics [16]-[17]. One of most important engineering applications is secure communication [18]-[19] because of the properties of random behaviors and sensitivity to initial conditions of chaotic systems.

In 1963, Lorenz, a meteorologist, studied a simplified model for thermal convection numerically. The model (today called the Lorenz model) consisted of a completely deterministic system of three nonlinearly coupled ordinary differential equations. He discovered that this simple deterministic system exhibited irregular fluctuations in its response without any external element of randomness being introduced into the system [20]. In 1976, O.E. Rössler constructed several three-dimensional quadratic autonomous chaotic systems, which also have seven terms on the right-hand side but with only one quadratic nonlinearity [21]. In 1994, J.C. Sprott suggests 19 cases of chaotic systems: case A-S with five linear terms and two nonlinear terms [22]. In 2000, J. C. Sprott found the functional form of three dimensional dynamical systems which exhibit chaos. Jerk equation has a simple nonlinear function, which can be implemented with an autonomous electronic circuit [23]. In 2010, J. C. Sprott gives Simple Autonomous chaotic circuit which employs an op-amp as a comparator to provide signum nonlinearity [24].

In this paper, in order to show some interesting phenomena of three dimensional autonomous ordinary differential equations, the chaotic behavior as a function of a variable control parameter a, has been studied. The complex dynamical behaviors of the system are further investigated by means of eigenvalue structures and various attractors. The chaotic system examined in MATLAB 2010. The Oscillator circuit of the chaotic system is afterwards designed by using MultiSIM software and a typical chaotic attractor is experimentally demonstrated.

2. Mathematical Model of the Jerk Circuit

Alpana Pandey modifies the system of equations Jerk into a system of simple quadratic equations. In this work, the Jerk circuit, which was firstly presented by Alpana Pandey in 2013 [25-26], is used. This is a three-dimensional autonomous nonlinear system that is described by the following system of ordinary differential equations:

$$\left.\begin{aligned}\dot{x} &= y\\ \dot{y} &= z\\ \dot{z} &= -x - y - az - bx^2\end{aligned}\right\} \tag{1}$$

The new system has one quadratic term and two positive real constants *a* and *b*. The parameters and initial conditions of the Jerk system (1) are chosen as: *a* = 0.54, *b* = 0.125 and (x_0, y_0, z_0) = (0.001, 0.01, 0.1), so that the system shows the expected chaotic behavior.

The equilibrium points of (1) denote by *E* ($\bar{x}, \bar{y}, \bar{z}$), are the zeros of its non-linear algebraic system which can be written as:

$$\left.\begin{aligned}0 &= y\\ 0 &= z\\ 0 &= -x - y - az - bx^2\end{aligned}\right\} \tag{2}$$

The Jerk system has two equilibrium points E_0 (1, 0, 0) and E_1 (−8, 0, 0). The dynamical behavior of equilibrium points can be studied by computing the eigenvalues of the Jacobian matrix *J* of system (1) where:

$$J(\bar{x}, \bar{y}, \bar{z}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1-0.250 * x & -1 & -0.54 \end{bmatrix} \tag{3}$$

For equilibrium points E_0 (1, 0, 0), the Jacobian becomes:

$$J(0,0,0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -0.54 \end{bmatrix} \tag{4}$$

The eigenvalues are obtained by solving the characteristic equation, $\det[\lambda I - J_1] = 0$ which is:

$$\lambda^3 + 0.54\lambda^2 + \lambda + 1 \tag{5}$$

Yielding eigenvalues of λ_1 = -0.816105, λ_2 = 0.138052 + 1.098304i, λ_3 = 0.138052 - 1.098304i, for *a* = 0.54, b = 0.125. For equilibrium points (-8, 0, 0), the Jacobian becomes:

$$J(-8,0,0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1000 & -1 & -0.54 \end{bmatrix} \quad (6)$$

The eigenvalues are obtained by solving the characteristic equation, $\det[\lambda I - J_1] = 0$ which is:

$$\lambda^3 + 0.54\lambda^2 + \lambda - 1000 \quad (7)$$

Yielding eigenvalues of λ_1 = 0.596175, λ_2 = -0.568087 + 1.163888i, λ_3 = -0.568087 - 1.163888i, for a = 0.54, b = 0.125. The above eigenvalues show that the system has an unstable spiral behavior. In this case, the phenomenon of chaos is presented.

3. Numerical Simulation Using MATLAB

In this section, we present numerical simulation to illustrate the dynamical behavior of Jerk circuit from system (1). For numerical simulation of chaotic system defined by a set of differential equation such as Jerk circuit, different integration techniques can be used in simulation tools. In the MATLAB 2010 numerical simulation, ODE45 solver yielding a fourth-order Runge-Kutta integration solution has been used. Figures 1(a)-(c) show the projections of the phase space orbit on to the *x–y* plane, the *y–z* plane and the *x–z* plane, respectively. As it is shown, for the chosen set of parameters and initial conditions, the Jerk system presents chaotic attractors of Rössler type.

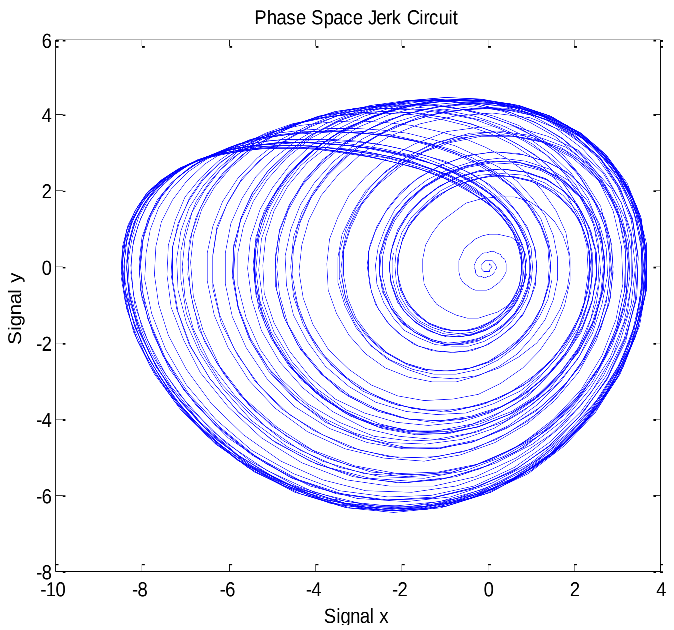
4. Analog Circuit Simulations Using MultiSIM

A simple electronic circuit is designed that can be used to study chaotic phenomena. The circuit employs simple electronic elements, such as resistors, capacitors, multiplier and operational amplifiers. In Figure 2, the voltages of C_1, C_2, C_3 are used as *x*, *y* and *z*, respectively. The nonlinear term of system (1) are implemented with the analog multiplier. The corresponding circuit equation can be described as:

$$\left.\begin{aligned} \dot{x} &= \frac{1}{C_1 R_1} y \\ \dot{y} &= \frac{1}{C_2 R_4} z \\ \dot{z} &= -\frac{1}{C_3 R_7} x - \frac{1}{C_3 R_8} y - \frac{1}{C_3 R_9} z - \frac{1}{10 C_3 R_{10}} x^2 \end{aligned}\right\} \quad (8)$$

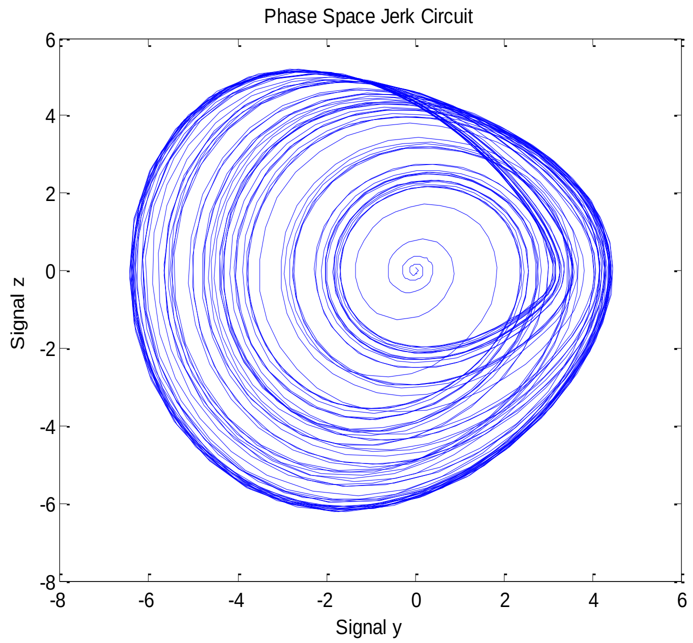
We choose $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = R_8 = 100$ kΩ, $R_9 = 200$ kΩ, $R_{10} = 80$ kΩ. $C_1 = C_2 = C_3$ $C_4 = 1$ nF. The circuit has three integrators, by using the Op-amp TL082CD, in a feedback loop and a multiplier (IC AD633). The supplies of all active devices ± 9 V. With MultiSIM 10.0, we obtain the experiment observations of system (1) in Figure 3. As compared with Figures 1 (a)-(c) good qualitative agreement between the numerical simulation and the MultiSIM 10.0 results of the Jerk circuit is confirmed. The parameter variable *a* of system (1) is changed by adjusting the resistor R_9, and obeys the following relations:

$$a = \frac{1}{C_3 R_9} \tag{9}$$

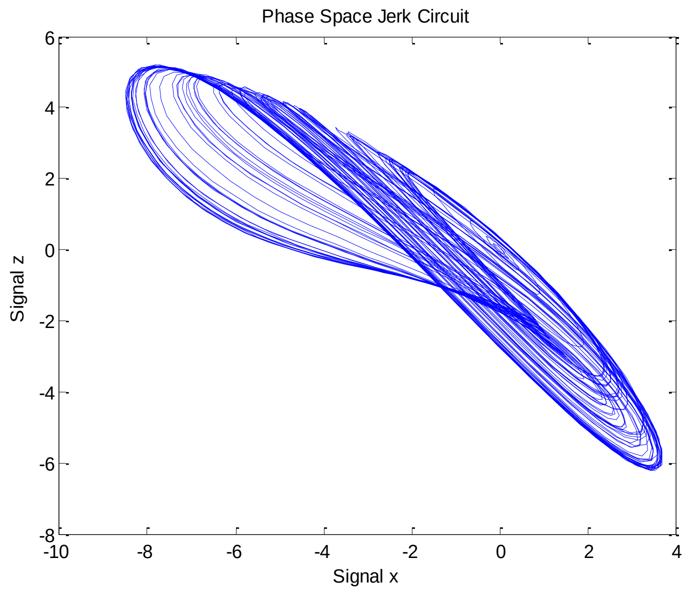


(a)

(continued)



(b)



(c)

Figure 1: Numerical simulation results using MATLAB 2010, for a = 0.54, b = 0.125, (a) x-y plane, (b) y-z plane, (c) x-z plane.

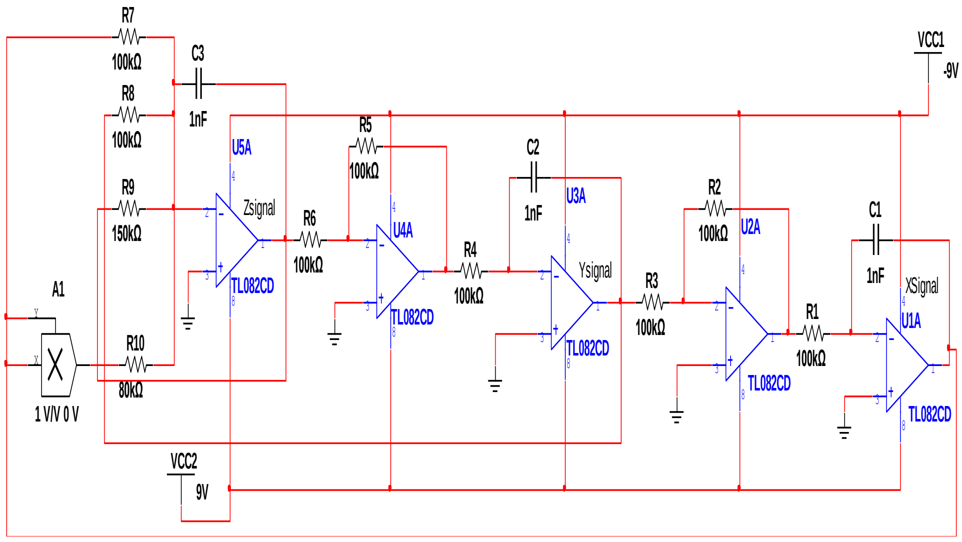
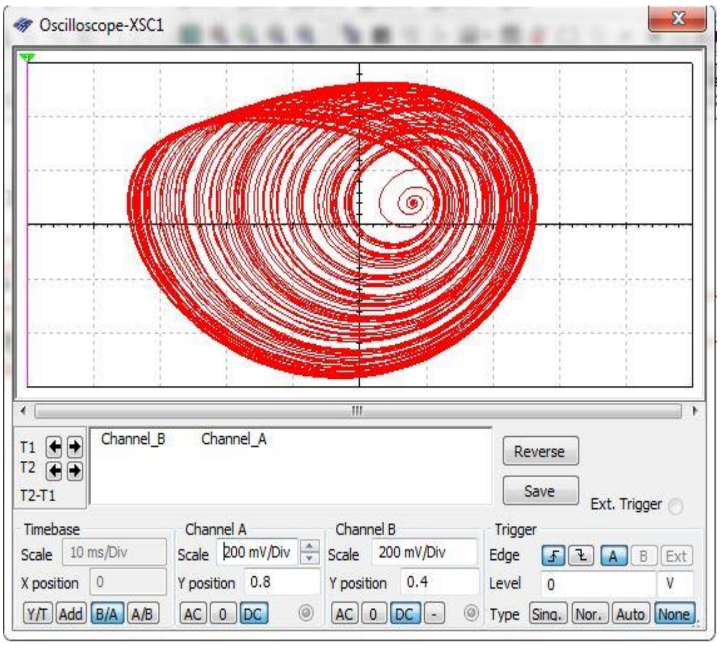
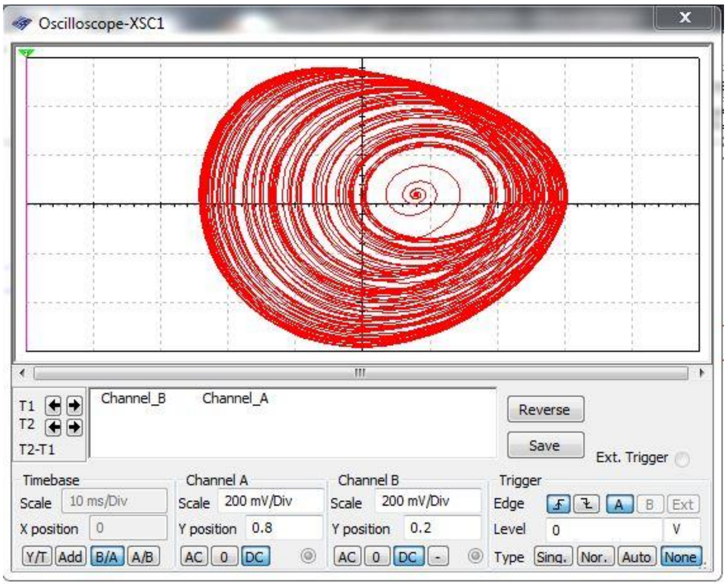


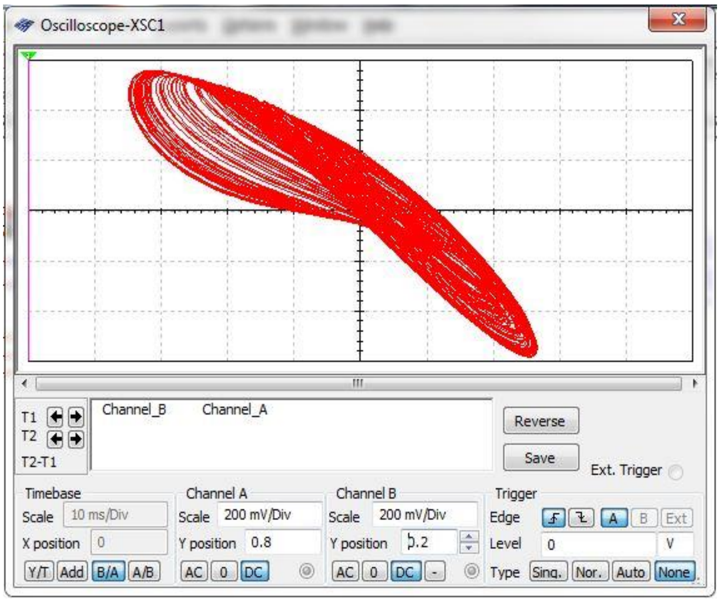
Figure 2: Schematic of the proposed Jerk circuit using MultiSIM 10.0.



(a)
(continued)



(b)



(c)

Figure 3: Various projections of the chaotic attractor using MultiSIM in
(a) *x-y* plane, (b) *x-z* plane and (c) *y-z* plane.

5. Circuit Realization of the Jerk Circuit

The chaotic dynamics of system (1) have also been realized by an electronic circuit. The designed circuitry realizing system (1) is shown in Figure 4. It consists of three channels to conduct the integration of the three state variable x, y and z, respectively. The operational amplifiers TL082CD and circuitry perform the basic operations of addition, subtraction and integration. The nonlinear term of system (1) are implemented with the analog multipliers AD633. We obtain the experimental observations of system (1) as shown in Figure 5. As compared with Figure 1 and Figure 3, a good qualitative agreement between the numerical and simulation results by using MATLAB 2010 and MultiSIM simulation with the experimental realization is confirmed.

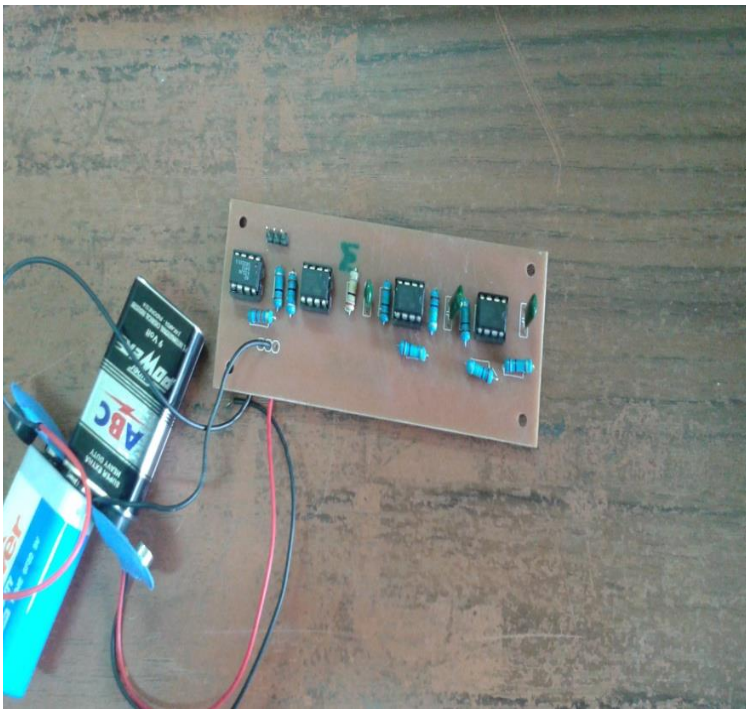
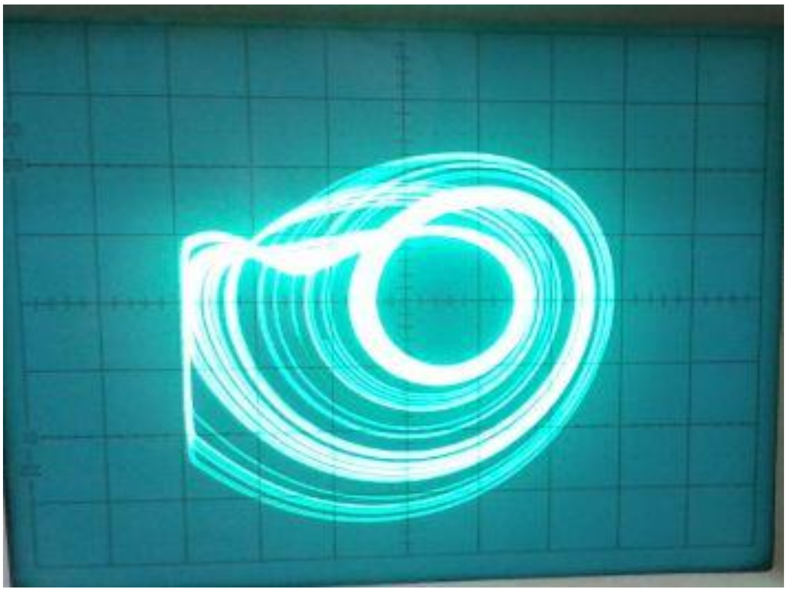
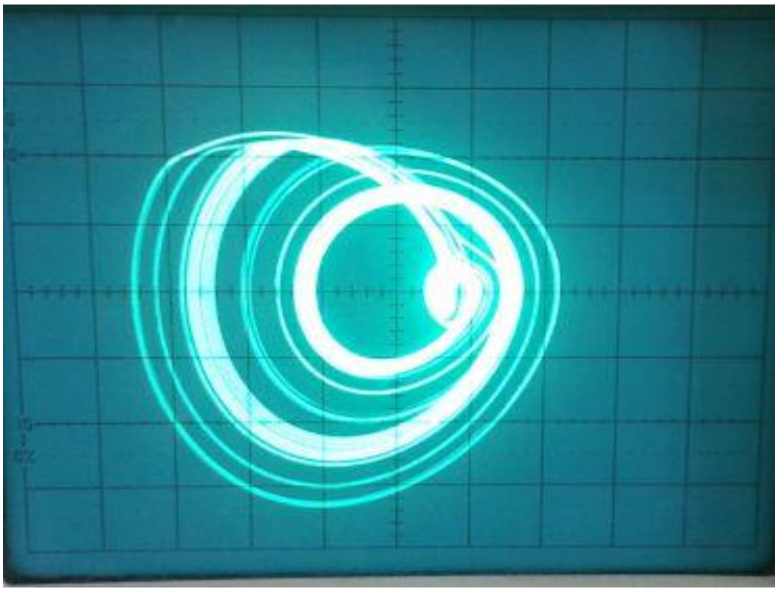


Figure 4: Electronic Circuit Realization for a = 0.5, b = 0.125

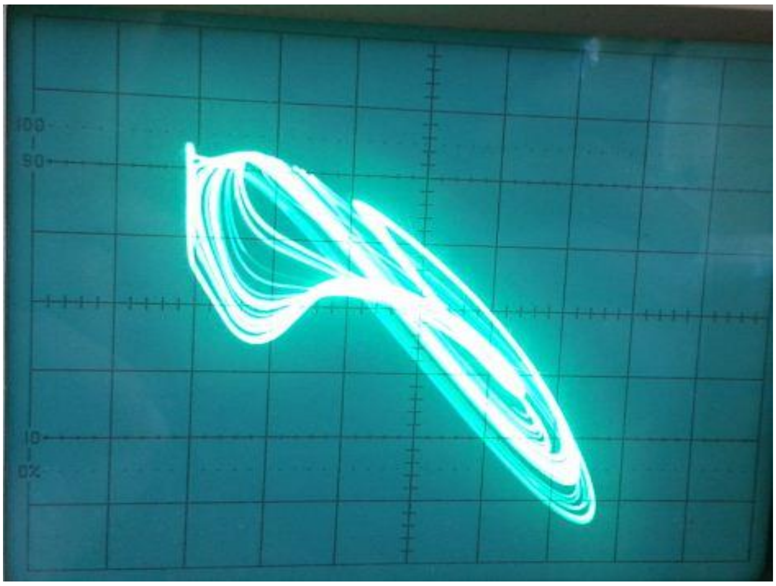


(a)



(b)

(continued)



(c)

Figure 5: Experimental Observations of the Chaotic Attractor in different planes (a) *x*-*y* plane (b) *x*-*z* plane (c) *y*-*z* plane for *a* = 0.54, *b* = 0.125

6. Conclusion

In this paper, we construct a three-dimensional autonomous system which has been rarely reported in three-dimensional autonomous systems in previous work. The system has rich chaotic dynamics behaviors. Moreover, it is implemented via a designed circuit with MultiSIM and tested experimentally in laboratory, showing very good agreement with the simulation result.

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