Fairness in Round Robin Tournaments

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1 Introduction

A single round robin tournament (RRT) based on a set T of teams is a schedule of matches where a match is a competition between two teams such that

- each team $i \in T$ plays against each other team $j \in T$, $j \neq i$, exactly once,
- each team does not play more often than once per period and
- the number of periods equals $|T| - 1$ and $|T|$ if $|T|$ is even and odd, respectively.

This structure can be arranged for each number of teams. Note that each team plays exactly once per period if $|T|$ is even and each team has exactly one period where is does not play if $|T|$ is odd. Among others, fairness is one of the major requirements in real world sports leagues as outlined in [Briskorn(2008)]. Several aspects of fairness in single RRTs are considered in the literature, e. g. carry-over effects in [Russell(1980)] and [Miyashiro and Matsui(2006)] and breaks in [de Werra(1982)] and [Post and Woeginger(2006)].

As proposed in [Bartsch(2001)] and [Briskorn(Forthcoming)] we consider a set S of strength groups being a partition $S = \left\{S_0, \ldots, S_{|S|-1}\right\}$ of T. A single RRT where

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no team plays against teams of the same strength group in two consecutive periods is called group-changing. Moreover, a single RRT where no team plays more than once against teams of the same strength group within $|S|$ consecutive periods is called group-balanced. [Briskorn(Forthcoming)] considers the case where $|T|$ is even and all strength groups have identical sizes, hence $|S_s| = \frac{|T|}{|S|}$, $s \in \{0, \ldots, |S|-1\}$. Construction schemes for group-changing single RRTs are given for each case except $|S| = 3$. Additionally, construction schemes for group-balanced single RRTs with $|S|$ even and $\frac{|T|}{|S|}$ even are given and it is proven that there is no group-balanced single RRT for all other cases.

The contribution of the submission at hand is twofold. First, we consider the case where $|S| = 3$ in section 2. Second, section 3 provides a construction scheme for group-balanced single RRT with an odd number of teams.

2 Three Groups

We consider the case where $\frac{|T|}{3} = 4k$, $k \in \mathbb{N}$. The basic idea is to schedule all matches between teams of identical strength groups in the set of periods

$$P' = \left\{3k - 1 \mid k \in \left\{1, \ldots, \frac{|T|}{|S|} - 1\right\}\right\}.$$

This can be done by scheduling a single RRT for each of the three groups since $\frac{|T|}{3}$ is even and $|P'| = \frac{|T|}{3} - 1$. Additionally, we consider the complete bipartite graph $K_{\frac{|T|}{3}, \frac{|T|}{3}}$ representing the set of matches to be scheduled for each pair of strength groups. It is well known to have a 1-factorization F^{bip}, as proposed for example in [de Werra(1980)]. Let $V_0 := \left\{i \mid i \in \left\{0, \ldots, \frac{|T|}{3} - 1\right\}\right\}$ and $V_1 := \left\{i \mid i \in \left\{\frac{|T|}{3}, \ldots, 2\frac{|T|}{3} - 1\right\}\right\}$ be the partition of the set of nodes of $K_{\frac{|T|}{3}, \frac{|T|}{3}}$. Then

$$F^{bip} = \left\{F_0^{bip}, \ldots, F_{\frac{|T|}{3}-1}^{bip}\right\}, \text{ where}$$
$$F_l^{bip} = \left\{\left[m, k + (m+l) \bmod \frac{|T|}{3}\right] \mid m \in \left\{0, \ldots, \frac{|T|}{3} - 1\right\}\right\}$$
$$\forall\, l \in \left\{0, \ldots, \frac{|T|}{3} - 1\right\}.$$

We will refer to two consecutive periods from $P \setminus P'$ as block in the following. For each pair of strength groups we arrange a specific 1-factor from F^{bip} in each block. We show that this can be done such that all matches between teams of different strength groups are arranged in periods $P \setminus P'$, such that each team plays exactly once in each period $p \in P \setminus P'$, and such that the resulting single RRT is group-changing.

3 Odd Numbers of Teams

If $|T|$ is odd then $|S|$ as well as $\frac{|T|}{|S|}$ is odd. We pick up the idea of pairings of strength groups as proposed in [Briskorn(Forthcoming)]. Analogously, we define a pairing of strength groups as a partition of strength groups into $\frac{|S|-1}{2}$ pairs of strength groups and a single strength group. We assign a pairing of strength groups to each period. Then, an assigned pairing of strength groups is interpreted as follows:

- if strength groups S_s and S_t, $s, t \in \{0, \ldots, |S|-1\}$, $t \neq s$, are paired in period $p \in P$ each team in S_s plays against a team in S_t in p,
- if strength group S_s, $s \in \{0, \ldots, |S|-1\}$, is not paired with an other strength group in period $p \in P$ teams in S_s play against each other (note that one team $i \in S_s$ does not play in p at all).

We propose a construction scheme for group-balanced single RRT for each odd $|T|$ and odd $|S|$ where $|T| = k|S|$, $k \in \mathbb{N}$, being divided into two steps:

1. arrange a pairing of strength groups in each period $p \in P$,
2. arrange matches in each period $p \in P$ based on the corresponding pairing of strength groups.

4 Future Work

There are several open questions regarding the existence of group-changing single RRT. We give a survey and first insights.

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