Assessing examination timetabling problems using fuzzy Pareto optimality

Taufiq Abdul Gani [1a]; Ahamad Tajudin Khader [2]; Rahmat Budiarto [3]; Mohammed Azmi Al-Betar [4]
School of Computer Sciences, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia [1,2,3,4]
{ taufiq; tajudin; rahmat; mohbetar}@cs.usm.my

1 Introduction

The nature of criteria or constraints is one of the issue that should be considered in solving examination timetabling problems. The constraints are often in conflicts where an improvement in one of them can only be achieved at the expense of worsening another. Students prefer longer breaks between two consecutive exams; however this preference is conflict with some pre-allocated timeslot requested by faculties. Therefore a single ideal or optimum solution that dominates the others is very difficult to find. Hence, a set of compromise solutions must be provided.

Pareto optimization (Coello 2003) has been well known as an approach to find a set of compromise solutions that represent a good approximation to the Pareto optimal front. The Pareto optimal front is the set of all non-dominated solutions in the multi-objective space. However, Farina and Amato (2004) argue that once the number of constraint increases, the Pareto approach is not appropriate anymore. It is because the current Pareto definition captures the notion of "optimality" in a narrowly prescribed sense. To overcome this drawback, they suggest to take into consideration the following factors: ***(i)*** the number of improved objectives, ***(ii)*** the size of such improvements and ***(iii)*** the decision maker's preferences between objectives (if any). To tackle those aspects they proposed fuzzy optimality as a combination of fuzzy logic (Zadeh 1965) and multi-objective optimization or multi-criteria decision-making.

Another main issue that has not been addressed adequately is to provide decision makers a framework to decide which non-dominated solutions appropriate to them. Recent assessment methodologies reviewed by Ruhul and Coello (2003) were only focus on descriptions of existences of non dominated solutions in the Pareto front. Hence a new assessment framework in understanding how the solutions evolve to better quality ones are still open for further studies.

In this paper, a previous work by Farina and Amato (2004) will be extended. This paper works on assesing the quality of non-dominated solutions of multi-objective examination timetabling problems. Fuzzy logic is applied here extensively. Fuzzy logic has been used to solve examination timetabling problems. Petrovic ***et al.*** (2005) use fuzzy logic to evaluate constraint violations, Asmuni ***et al.*** (2005) used fuzzy logic to select heuristic ordering methods in constructing examination timetabling, then Asmuni ***et al.*** (2006) try to asses the quality of timetabling using fuzzy logic. In contrast to them, this paper uses fuzzy

[a] Corresponding author.
Email addresses: taufiq@cs.usm.my, topgan@gmail.com.
(✉): School of Computer Sciences, Universiti Sains Malaysia,11800 USM, Pinang, Malaysia.
Tel. No :+6281360436282.

logic both to model constraints and to evaluate the quality of examination timetabling solutions using fuzzy Pareto dominance or the size of constraint violation's changes.

2 A fuzzy model for multi-objective examination timetabling problems

The constraints of timetabling problems are divided into hard and soft categories based on a distinction between feasible and (near) optimal timetables. Hard constraints are those that must be fulfilled, while soft constraints are those that are desirable but not absolutely essential. Burke and Petrovic (2002) have presented some common soft constraints for university timetabling. The constraints may be conflicting in the sense that an attempt to satisfy one of the constraints can lead to the violations of another as explained in Section 1.

In the following, three constraints are modeled as follows:

- *Spreading events out in time(C_1);* The number of common students seating on both exam *x* and *y* which are scheduled within two consecutive time slots will be calculated and penalized. Afterwards C_1 are calculated by summing up the pinalty values and divided by the total number of enrollments.

- *Time assignment (C_2);* Exams may be expected to be scheduled at certain timeslots. The violations of these constraints will be penalized. C_2 is calculated by summing up the number of violations then normalized (divided) by number of exams.

- *Earlier timeslot for large-sized exams (C_3);* Large-sized exams are expected to be scheduled in earlier time slot. Two variables such as *time slot* and *exam size* must be considered in evaluating this constraint. The problems, it is very difficult or vague to derive a mathematical model precisely for these two variables. This phenomenon (vagueness and imprecision) are called fuzziness (Zadeh 1965).

This constraint (C_3) is modeled using fuzzy approach. Two linguistic variables such as *Slot* for time slot and *Size* for exam size are used. *Slot* has value as members of fuzzy set = *{early, middle, late}*. Base values for each member are in the scale of *0* to *1*. The earliest or the first time slot is given as *0* where the latest is *1*.

Size has value as members of fuzzy set = {*small, medium, large*}. Base values for each member are in the scale of *0* to *1*. The values are obtained by normalized the actual exam size (number of students taken the exams). *0* is given the smallest exam size and *1* is for the largest.

Whether scheduling an exam to a certain timeslot has satisfied constraint C_3 or not is measured by another linguistic variable called *Fitness*. This variable has values as member of fuzzy set = *{slow, medium, high}*. Base values for each member are in the scale of *0* to *1*. *0* is given for the inappropriate timeslot allocation and *1* is for the appropriate. The fuzzy rules for C_3 constraints will assign the fitness to be higher if the constraint can be fulfilled.

3 Fuzzy Pareto Dominance and Fitness Calculation

The number and size of improvement of each objective is not considered in Pareto-optimality (Coello 2003). In Fuzzy Pareto Dominance (Farina and Amato 2004) the size of improvement n_e, n_b and n_w (*e* for equal, *b* for better, *w* for worst)is obtained by taking differences of objective functions. The differences are given to membership function μ_e, μ_b, μ_w. Given two vector *v1* and *v2*, and *M* of fitness or evaluation fuction, *f*. n_b is calculated using Equation 1.

$$n_b(v_1, v_2) = \sum_{i=1}^{M} \mu_b(f_i(v_1) - f_i(v_2)) \qquad (1)$$

n_e and n_w are calculated in the same way using their appropriate membership function μ_e, μ_w.

The calculation of size of improvement between two given solutions (Equation 1) is extended to calculate the fitness of a given solution relative to the rest of solutions in a population. This fitness is calculated as follows; ***(i)*** averaging all the size of improvement (n_b, n_e, n_w) of a given solution to each of the other solutions. ***(ii)*** the value of the fitness is calculated based on Fuzzy Rules in such a way if the size of impromens dominate the size of worsen, the membership value of the fitness will be higher.

4 Computational Experiment and Result

The concept of fuzzy Pareto dominance and the calculation of fitness evaluation proposed are applied to assess the quality of the fittest non-dominated solutions. Data test used in these experiments are taken from a real case of Universiti Sains Malaysia. The problem size, the number of students, exams, enrollments and periods are 15015, 574, 63880 and 40 respectively. An Evalutionary Algorithm was developed incorporating the proposed Fuzzy Pareto Dominance and fitness calculation. To asses quality of non-dominated solutions, in each generation these parameters such as: ***(i)*** Objective values of each constraints; ***(ii)*** Fitness values; ***(iii)*** Size of improvement are recorded. The results are discussed as follows:

- *Constraint Violations***:** Three constraint models (C_1, C_2 *and* C_3) were introduced. C_1 and C_2 are expected to be minimized while C_3 to be maximized. In Figure 1, the averages of C_1, C_2 *and* C_3 of the fittest of non-dominated solutions are presented. C_1 and C_2 decreased which mean they were improved, and C_1 was improved more than C_2. However the great improvements of C_1 compensates to the worsening of C_3. The spread of the exams (C_1) conflicts the alocation of large exams in earlier timeslot .

- *Fitness Values***:** The fitness values in Figure 2 shows that in overall, the quality of the fittest solutions has improved. Along the generations, the differences of fitness between the fittest solutions and their associate solutions in the population increase. It means that the gap of fittest of the fittest solution with its associates in the population is wider.

- *Size of Improvements:* The improvements of the fittest solutions were shown in Figure 1 and 2. How the improvements in each of generation is presented in Figure 2. Along the execution, the equality (N_e) of constraint violations decreases. Since $N_e+N_b+N_w$ is equal to 1. The decrements of N_e compensate to the changes

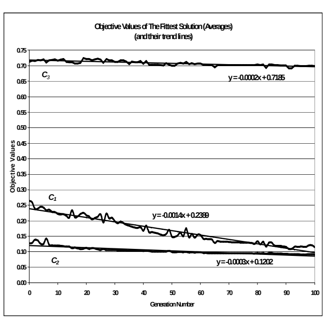

Figure 1 Objective Value of the Fittest Solution

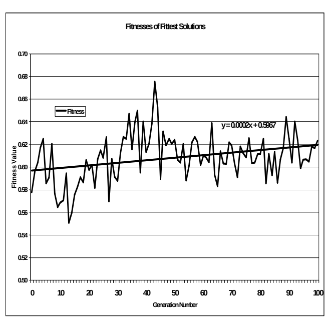

Figure 2 Fitness of The fittest Solutions

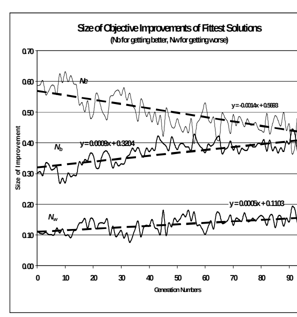

Figure 3 Sizes of Improvement

both N_w and N_b. Both values are getting higher, however, N_b (for changing to better one) is higher than N_w (the worst one).

5 Conclusion

This paper focused on evaluations of the fittest non-dominated solutions in conflict and multi-objective examination timetabling problems. Concepts of fuzzy optimality in determining dominance relationships between two given solutions have been extended. In contrast to assessment approaches usually found in timetabling researches, the proposed approach is able to capture individual behavior of the constraints of the solutions during their evolutions. A graph of size of improvement illuterates how conflicting constraints are compensated. Convergences of quality of multi-objective timetabling solutions are described more clearly.

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