

## Application Research of Wavelet Fusion Algorithm in Electrical Capacitance Tomography

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**Abstract.** Electrical capacitance tomography (ECT) technology has the soft-field nature and the ill-posed problems. To solve the problem of low imaging quality by standard Tikhonov regularization, a wavelet fusion algorithm based on Tikhonov regularization is proposed in this paper. Firstly, Tikhonov regularization method is used to solve ill-posed characteristic of system inverse problem, then the initial image is decomposed and fused by using wavelet transform. The simulation results show that this method can not only improve the image reconstructed by standard Tikhonov regularization, but also provides a new idea to ECT image reconstruction.

**Keywords:** ECT system; standard Tikhonov regularization; wavelet decomposition; image fusion

### 1 Introduction

There are difficulties to reconstruct image for ECT system because of the soft field characteristic of the ECT's capacitive sensor and the equation of ECT's forward problem is morbid. Tikhonov regularization algorithm can efficiently solve the ill-posed problems, which is widely used in image reconstruction of ECT in recent years. The regularization parameters choice is the key for Tikhonov regularization algorithm. The usual practice is iterative method. But iterative method has many shortcomings. In this paper, to be aimed at the shortcomings of iterative method, a new processing method is proposed. The new method is wavelet fusion algorithm which based on Tikhonov regularization.

### 2 ECT Imaging Principle Based on Tikhonov Regularization Algorithm

The ECT system is composed of three parts: capacitive sensor, data acquisition system and imaging computer. The measured capacitance value will change with the

change of the distribution of medium in the pipeline. So we can obtain media distribution according to the actual value of the measured capacitance [1].

Represents the ECT system model of direct problems with matrix as follows[2].

$$C=SG \tag{1}$$

Among formula (1),  $C$  is normalized capacitance vector,  $S$  is normalized sensitivity matrix and  $G$  is permittivity. Permittivity represents the image gray value in the image reconstruction.

The inverse problem of ECT system is to reconstruct the distribution of permittivity in medium by measuring data (capacitances). Tikhonov regularization algorithm is a typical representation. The solution of the standard Tikhonov regularization algorithm can be expressed as follows [3].

$$G = S^T S a I + S C . \tag{2}$$

However, the image edges reconstructed by Tikhonov regularization algorithm are irregular. Aiming at this question, this paper introduces wavelet decomposition which is used to fusion images and can improve the imaging quality of the ECT system.

### 3 The Wavelet Fusion Algorithm Based on Tikhonov Regularization

In the image reconstruction, the boundary quality needs to be further improved [4]. Our algorithm firstly decomposes the images reconstructed by Tikhonov regularization algorithm using wavelet, and then fusion the processed images.

#### 3.1 Image Decomposition with Wavelet

The image is different from the one-dimensional signal, and any point  $(x, y)$  has a gray value of image counterpart [5]. When image's coordinate continuously change, it can be defined as a continuous change of two-dimensional signal  $f(x, y)$ . In order to analyze the details and edge of the image reconstructed by Tikhonov regularization algorithm, we need to carry on multi-resolution decomposition processing to the image. The formula is summarized as follows when we are doing  $j$  order analysis [6].

$$\left. \begin{aligned} & \int_{j k_1}^{j k_2} A f(x_1, x_2) = f(x_1, x_2), \Pi(x_1) \Pi(x_2) \\ & (1) \\ & \int_{j k_1}^{j k_2} D f(x_1, x_2) = f(x_1, x_2), \Pi(x_1) \Pi(x_2) \end{aligned} \right\} \tag{3}$$

$$\int_{j k_1}^{j k_2} D f(x_1, x_2) = f(x_1, x_2), \Pi(x_1) \Pi(x_2) \tag{2}$$

$$(3) (\cdot, \cdot) = \int \mathcal{D}f x x f x x$$

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$$(\cdot, \cdot), \mathcal{J}(x) \mathcal{J}(x) \\ \substack{1 \\ 12, jk 1jk 2}$$

### 3.2 ECT Image Fusion Method Based on Wavelet Decomposition

After having decomposed the image by wavelet, we select the characteristics on transform domain, and then create fused images.

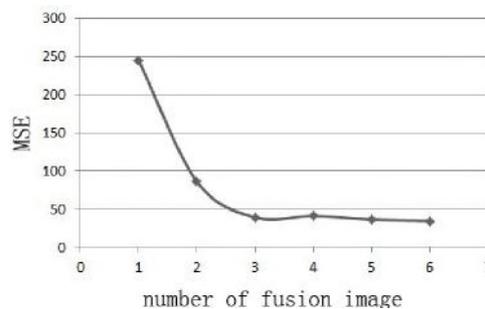
Low frequency component represents the general trend of the image, and expresses the central area of the reconstructed image[7]. We use the pixel gray value selection algorithm to fuse low frequency region. The pixel gray value selection algorithm includes maximum and minimum pixel gray value selection algorithm. Suppose that we use  $P_n(x, y)$  ( $n=1, 2, 3$ ) to represent the original images. The size of the image is  $M*N$  pixel, and  $R(x, y)$  represents the fused image. The maximum pixel gray value selection algorithm can be expressed as follows.

$$R(x_i, y_j) = \max(P_1(x_i, y_j), P_2(x_i, y_j), P_3(x_i, y_j)) \quad i = 1, 2, \dots, M; j = 1, 2, \dots, N. \quad (4)$$

we use the maximum pixel gray value selection algorithm to fusion the low frequency component so that it can strengthen the display effect of central area. The high frequency component represents the edge of image whose brightness is large variable. For the algorithm to fusion the edge of the image, we need to calculate the mean of edge of three images so that it can reduce error. So we choose the average gray value algorithm. It can be express as follows.

$$R(x_i, y_j) = (P_1(x_i, y_j) + P_2(x_i, y_j) + P_3(x_i, y_j)) / 3 \quad i = 1, 2, \dots, M; j = 1, 2, \dots, N. \quad (5)$$

We use Mean Square Error(MSE) to evaluate the image quality while the number of images changes. The curve of MSE is shown in Figure 1.



**Fig. 1.** Relationship curve of fusion image number and mean square error.

From the Figure 1, it can be seen that mean square error tends to be stable from the 3rd fusion image. So to fuse 3 images are the best.

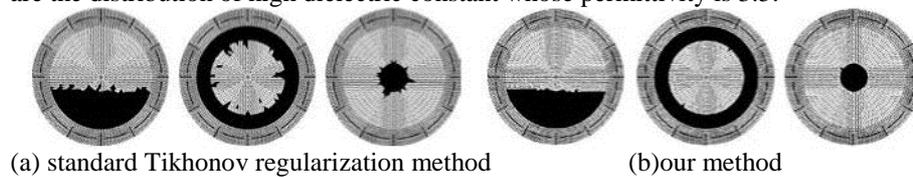
The steps of the image fusion are shown as follows.

1. Decompose the three images with selected wavelet function and then construct wavelet pyramid image.
2. Fuse the high frequency component and low frequency component by using fusion method in the formula, and get fused wavelet pyramid image.
3. Process inverse transform on the fused image and get the final image.

## 4 Data Simulation and Result Analysis

In order to verify the validity of the results, we carried out simulation for the three most common flow patterns of ECT (circular flow, central flow and laminar flow) with the ANSYS environment.

The simulation was carried out with 12-electrode ECT system. The image reconstructed by our method was compared with that reconstructed by standard Tikhonov regularization. Results are shown respectively in Figure 2. The white parts are the distribution of low dielectric constant whose permittivity is 1. The black parts are the distribution of high dielectric constant whose permittivity is 3.5.



**Fig. 2.** The image reconstructed by standard Tikhonov regularization method and our method.

## 5 The conclusion

This paper gives a wavelet fusion algorithm based on Tikhonov regularization, and the simulation experiment is given in the end. Compared with standard Tikhonov regularization, the experimental results show that the wavelet fusion algorithm based on Tikhonov regularization can improve the image quality and has high application value, but the algorithm has a higher computational complexity because of the wavelet decomposition and image fusion technology. Therefore, the algorithm fits the application where lower speed is required. It is expected that the study of the wavelet fusion algorithm based on Tikhonov regularization will focus on improving the speed of wavelet transformation and image fusion.

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