

Influence of the Depth of Long-Wavelength Defects on the Actions on a Railway Track

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Abstract. The defects with long wavelength, which play a key role on track deflection and the loads that develop, are analyzed using the second order differential equation of motion. The dynamic stiffness coefficient of the track, as well as the ratio between the period of the acting load (forcing period) compared to the eigenperiod of the track are of great importance, for the forces that are developed on track.

Keywords: railway track; dynamic stiffness; actions/ loads; deflection; subsidence; eigenperiod; forcing period.

1 Introduction

The railway track is usually modeled as a continuous beam on elastic support. Train circulation is a random dynamic phenomenon and, depending on the different frequencies of the loads it imposes, there is a corresponding response of the track superstructure. The dynamic component of the load is primarily caused by the motion of the vehicle's Non-Suspended (Unsprung) Masses, which are excited by track geometry defects, and, to a smaller degree, by the effect of the Suspended (sprung) Masses. The statistical probability of exceeding the calculated load -in real conditions- should be considered, so that the corresponding equations would refer to the standard deviation (variance) of the load.

2 Static Component of the Actions

The most widely used theory (referred to as the Zimmermann theory) based on Winkler analysis examines the track as a beam on elastic support.

$$(1) \quad \frac{d^2 y}{dx^2} = -\frac{M}{EJ}$$

where y is the deflection of the rail, M is the bending moment, J is the moment of inertia of the rail, and E is the modulus of elasticity of the rail. From the formula above it is derived that the reaction of a sleeper R_{static} is:

$$(2) \quad R_{stat} = \frac{Q_{wheel} \ell^3}{2 E J} \quad \text{where } Q_{wheel} \text{ is the static wheel load, } \ell \text{ the distance between sleepers, } E \text{ and } J \text{ the modulus of elasticity and the moment of inertia of the rail, } R_{stat} \text{ the static reaction/action on the sleeper, and } \rho \text{ reaction coefficient of the sleeper which is defined as: } \rho = R/y, \text{ and is a quasi-coefficient of the track elasticity (stiffness) or a spring constant of the track. The track consists of a sequence of materials (substructure, ballast, sleeper, elastic pad/fastening, rail), that are characterized by their individual coefficients of elasticity (static stiffness coefficients) } \rho_i.$$

Hence, for $\frac{1}{\rho_{total}} = \sum_{i=1}^v \frac{1}{\rho_i}$ the track:

$$(3) \quad \text{where } v \text{ is the number of various layers of materials that exist under the rail -including rail- elastic pad, sleeper, ballast etc. The semi-static Action/Reaction is produced by the centrifugal acceleration exerted on the wheels of a vehicle that is running in a curve with cant deficiency, given by: } Q_{hcg} = \frac{V^2 \alpha}{g e} \text{ where } \alpha \text{ is the cant deficiency, } Q_{hcg} \text{ the height of the center of gravity of the vehicle from the rail and } e \text{ the gauge.}$$

3 Dynamic Component of the Actions: General Solution

The dynamic component of the acting load consists of the action due to the Suspended Masses (SM) and the action due to the Non Suspended Masses (NSM) of the vehicle. To the latter a section of the track mass is added, that participates in its motion ([1], [2]). Based on a cosine defect:

3 Dynamic Component of the Actions: General Solution

(4)
$$y = a \cos \left(\frac{V}{L} t \right)$$

The second order differential equation of motion is:

$$(5) \quad m_{NSM} \frac{d^2 z}{dt^2} + \zeta \frac{dz}{dt} + h_{TRACK} z = m_{NSM} a \cos \left(\frac{V}{L} t \right)$$

The complete solution of which using polar coordinates is ([2], p.199 and ch.3):

$$(6) \quad z = \underbrace{A e^{-\zeta t}}_{\text{transient part}} + \underbrace{B \cos \left(\frac{V}{L} t \right) + C \sin \left(\frac{V}{L} t \right)}_{\text{steady state part}}$$

where, the first term is the transient part and the second part is the steady state [4].

4 Analysis for an Isolated Defect, Forcing vs Eigen Period

(7) We focus herein on the term $A e^{-\zeta t} \sin \left(\frac{V}{L} t \right)$ from Equation 6 which represents the transient part of motion. We investigate this term for $\zeta=0$.

$NSM \oplus \frac{TRACK}{dt} NSM$

(

$NSM \oplus \frac{TRACK}{dt} NSM \oplus m = g$

Where: g the acceleration of gravity,
$$h_{TRACK} = 2.2 \oplus \frac{EJ}{\ell^3}, \text{ where } m_{TRACK} = 2.2 m_{NSM} \oplus m_{total} \quad (8)$$

ρ_{total} the total static stiffness coefficient of the track, ℓ the distance among the sleepers, E, J modulus of elasticity and moment of inertia of the rail, for comparison of m_{TRACK} to measurements see [5]. The general solution of equation (7) is:

$$(9) \quad z(t) = \left[\frac{1}{2} \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_n} t \right) + \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_1} t \right) \right] \cos \left(\frac{2\pi}{T_n} t \right) + \left[\frac{1}{2} \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_n} t \right) + \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_1} t \right) \right] \sin \left(\frac{2\pi}{T_n} t \right) \quad (10)$$

where, $T_n = 2\pi/\omega_n$ the period of the free oscillation of the wheel circulating on the rail and $T_1 = 2\pi/\omega_1$ the necessary time for the wheel to run over a defect of wavelength X : $T_1 = X/V$. Consequently, $T_n/T_1 = \omega_1/\omega_n$. From equation (9):

$$(10) \quad z(t) = \left[\frac{1}{2} \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_n} t \right) + \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_1} t \right) \right] \cos \left(\frac{2\pi}{T_n} t \right) + \left[\frac{1}{2} \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_n} t \right) + \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_1} t \right) \right] \sin \left(\frac{2\pi}{T_n} t \right)$$

where $n = \omega_n/\omega_1$, $\omega_1 = X/V$ and we examine values of $\mu \cdot X = 0, 0.1X, 0.2X, \dots, 0.8X, 0.9X, X$. for discrete values of $n = \omega_n/\omega_1 (= T_1/T_n)$ and μ a percentage of the wavelength X . In Figure 1 Left the equation (10) is depicted.

5 The Depth of a Long-Wavelength Defect and the Implied Actions on a High Speed Railway Track

The first term in the bracket of equation (10) is depicted on the vertical axis while on the horizontal axis the percentages of the wavelength $\mu \cdot X$ are shown. We observe that $z(x)$ has its maximum value for $T_1/T_n = 0.666667 = 2/3$, equal to 1.465:

$$z(t) = \left[\frac{1}{2} \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_n} t \right) + \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_1} t \right) \right] \cos \left(\frac{2\pi}{T_n} t \right) + \left[\frac{1}{2} \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_n} t \right) + \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_1} t \right) \right] \sin \left(\frac{2\pi}{T_n} t \right) \quad (11)$$

The relation T_1/T_n represents the cases for short and long wavelength of the defects. For $T_1/T_n = 2-2.5$ the wavelength is long and for T_1/T_n the wavelength is short ([3], p.49). The second derivative of $z(x)$ from equation (11), that is the vertical

T_1/T_n	2,5	2	1,5	1	0,8	0,6667	0,6	0,5
ζ	0,18	0,335	0,65	1,205	1,415	1,47	1,43	1,34

where: $\zeta = [(m_{NSM} + m_{TRACK})/m_{NSM}] \cdot [Z_{max}/\alpha]$

acceleration that gives the dynamic overloading due to the defect, is calculated:

$$(11) \quad z(t) = \left[\frac{1}{2} \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_n} t \right) + \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_1} t \right) \right] \cos \left(\frac{2\pi}{T_n} t \right) + \left[\frac{1}{2} \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_n} t \right) + \left(\frac{m_{NSM}}{m_{NSM} + m_{TRACK}} \right) \cos \left(\frac{2\pi}{T_1} t \right) \right] \sin \left(\frac{2\pi}{T_n} t \right)$$

$$(12) \quad \left[\frac{m_{NSM} + m_{TRACK}}{m_{NSM}} \right] \left[\frac{1}{\gamma} \right] \left[\frac{1}{\tau} \right] \left[\frac{1}{\omega_1} \right] \left[\frac{1}{\omega_n} \right] \left[\frac{1}{\alpha} \right] \left[\frac{1}{\lambda} \right] \left[\frac{1}{T_n} \right] \left[\frac{1}{T_1} \right] \left[\frac{1}{T} \right] \left[\frac{1}{\omega} \right] \left[\frac{1}{\omega_0} \right] \left[\frac{1}{\omega_c} \right] \left[\frac{1}{\omega_d} \right] \left[\frac{1}{\omega_e} \right] \left[\frac{1}{\omega_f} \right] \left[\frac{1}{\omega_g} \right] \left[\frac{1}{\omega_h} \right] \left[\frac{1}{\omega_i} \right] \left[\frac{1}{\omega_j} \right] \left[\frac{1}{\omega_k} \right] \left[\frac{1}{\omega_l} \right] \left[\frac{1}{\omega_m} \right] \left[\frac{1}{\omega_n} \right] \left[\frac{1}{\omega_o} \right] \left[\frac{1}{\omega_p} \right] \left[\frac{1}{\omega_q} \right] \left[\frac{1}{\omega_r} \right] \left[\frac{1}{\omega_s} \right] \left[\frac{1}{\omega_t} \right] \left[\frac{1}{\omega_u} \right] \left[\frac{1}{\omega_v} \right] \left[\frac{1}{\omega_w} \right] \left[\frac{1}{\omega_x} \right] \left[\frac{1}{\omega_y} \right] \left[\frac{1}{\omega_z} \right]$$

for discrete values of $n = \omega_n / \omega_1 (= T_1 / T_n)$ and μ a percentage of the wavelength X , and $T_n = 0,0307$ sec as calculated above. The maximum value of z is given in Table 1 below, as it is –graphically– measured in Figure 1 Left (the damping was omitted).
Table 1: Maximum Values of $\zeta = [(m_{NSM} + m_{TRACK}) / m_{NSM}] \cdot [z_{max} / \alpha]$

As a case study we use a ballasted track, for high speed, equipped with rail UIC60, sleepers B70 type, W14 fastenings, ballast fouled after 2 years, subgrade for high speed lines, $h_{TRACK}=8539,6t/m$, $m_{TRACK} = 0,426 t$, $m_{NSM}=1 t$.

$$z = \frac{112.500N}{22 EJ} \frac{600^3 mm^3}{210.000 \frac{N}{mm} \oplus 3,06 \oplus 10 mm \oplus 85.396 \frac{N}{mm^2}} \frac{112.500N}{22} \frac{1,524228617 \cdot 10^{-5} m}{N} = 0,606 mm$$

For $\alpha=1 mm$, the dynamic increment of the static deflection is equal to $(0,133/0,606)=21,9\%$ of the static deflection (for every mm of the depth of the defect). If we examine the second derivative (vertical acceleration) a percentage of g:

$$z = \frac{1}{2} \left(\frac{t}{\lambda} \right)^2 - \frac{1}{\lambda} \left[\frac{f_A}{L V} \frac{\alpha \oplus \lambda}{\cos \nabla} \frac{2 \nabla \alpha \oplus \lambda}{A V} \frac{2}{n} \Phi_{,2,0}(n)^2 \cos(2n \nabla \alpha \oplus \lambda) \right] \quad [\%g]$$

{ steady state transient part }

Equation (13) is plotted in Figure 1 Right.

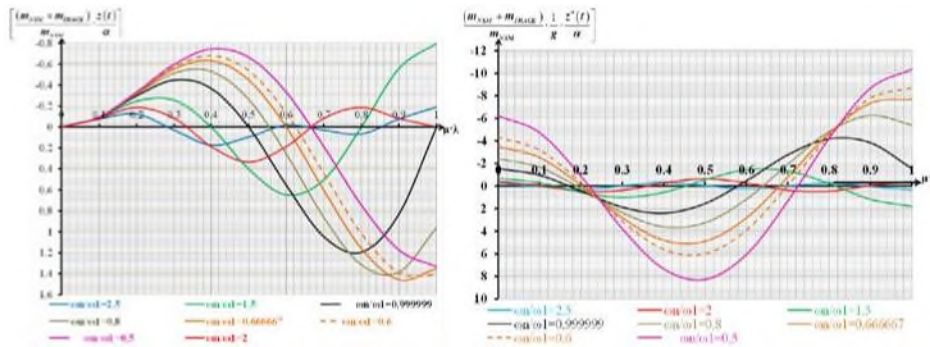


Fig. 1. (Left) Mapping of Equation (9), in the vertical axis the first term of equation (10), in the horizontal axis the percentage of the wavelength λ of the defect are depicted.

6 Conclusions

For a defect of wavelength λ and sagitta of 1 mm (depth of the defect), the dynamic increase of the acting load –compared to the static wheel load– is equal to 9,24%. When the speed increases, the period T_1 decreases and the supplementary sagitta (depth of the defect) increases. The increase of the dynamic component of the load increases faster since it is dependent on the square of the speed $(\omega_1)^2$. When the dynamic stiffness coefficient h_{TRACK} increases, T_n decreases, T_1/T_n increases, the supplementary sagitta decreases (for the same V), and the dynamic component of the action decreases also.

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