

# Analysis the effect of phase errors on angle estimation performance in monostatic multiradar networks

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**Abstract.** Angle estimation is one of the main tasks of most radar systems. In this paper we have investigated phase errors in an incident signals with different kinds of noise distributions and variances. Their effects on final angle estimation were simulated by means of computers simulations. Based on our study we have derived recommendation for different noise environments.

**Keywords:** monostatic radar, angle error analysis, phase errors, multiradar.

## 1. Introduction

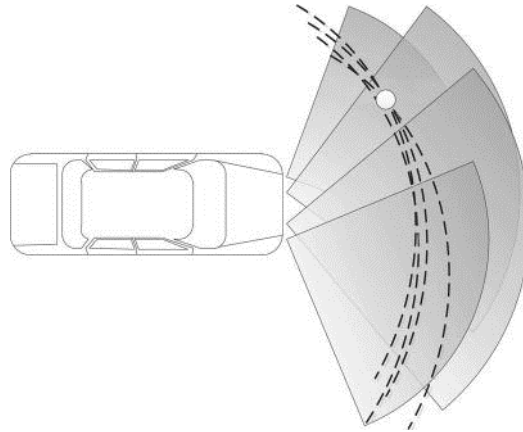
With recent development of radar technologies for public safety and automotive applications, the need for fine angle estimation in multipath and multitarget situations became very important [1]. There are many automotive applications like adaptive cruise control, Stop&Go, PreCrash, or Parking Aid which depend on accurate angle estimation in order to locate the objects on the scene. One of the simplest approaches for practical radar systems for automotive applications was developed in [2], [3]. They have constructed multilateration based multi-radar network. Multilateration radars only provide range measurements which are then used to locate the targets. This technique is popular in outdoor and indoor positioning systems [4].

One of the important issues, the effect of phase error on angle estimation of current radar systems is not well presented in literature, especially for multilateration radar networks. The main objective of the paper is to investigate how the final angle estimate affected by phase errors in an incident signals. For the analysis we have chosen the practical systems which are implemented and tested in real tests in [2], [3].

The paper is organized as follows: in section 2 we will introduce system architecture and multilateration methods. Section 3 will introduce system simulation study and analysis of phase errors on angle estimation. Finally section 4 will introduce concluding remarks and recommendations.

## 2. System model and related works

The multilateration based radar network consists of several monostatic radars which are capable of measuring accurate distances to the target. As they operate at 24 GHz and 77 GHz [2], [3], the achievable accuracy is within several centimeters. They can be placed all around the vehicle based on applications. In our study we assume they are placed in front bumper of the car as shown in Fig. 1.



**Fig. 1.** Car equipped with 4 monostatic radar network.

Each radar sensor is capable of measuring accurate range between target and the sensor. After all sensors in the network perform range measurements, it is possible to calculate the location of the target in the area of interest by means of multilateration technique. Mainly there are many techniques available in literature to calculate the position of the target [5]. One of the widely used techniques is least square (LS) based methods.

A LS position location for a single target is given by:

$$Y_{tg}$$

Sensors' position  $s_1, s_2, s_3, s_4$  on the front bumper of the car and their differences  $i_1, I_2, I_3, I_4$  are given by:

Estimated target position is:  $t_g = [x_{tg}]$

$$s_i = [x_i, y_i]^T, \quad l_i = s_i - t_g = [x_i - x_{tg}, y_i - y_{tg}]^T, \quad i = 1, \dots, 4$$

The measured ranges at each sensor are:

$$r_{oi} = \sqrt{(x_i - x_{tg})^2 + (y_i - y_{tg})^2} + n_i, \quad i = 1, \dots, 4$$

where  $n_i$  is additive noise to the measured distance. The description of the noise term is discussed in detail in next section.

The position of the target can be calculated using set of equations in Session 6A 713

(1). But due to non-linear term in the equation set, it is difficult to solve it. Around the estimated

target's position  $tg^{(0)} = \begin{matrix} [x, \\ y] \end{matrix}$ , the non-linear equations have to be linearized. One

of the simplest and fast convergence iterative linearization methods is iterative LS algorithm [2], [6]. Then iterative LS algorithm applied to set of equations based on (1), as was done in [2]:

$$r_{mi} = \frac{a(r^{(0)})}{a(x_{ig}, y_{ig})} (y_{ig} - y_i^{(i)}) + F(e) \quad (2)$$

The linearized measurement equations are now:

$$VZ = H VZ$$

$$\begin{pmatrix} r_{m1} \\ r_{m2} \\ r_{m3} \\ r_{m4} \end{pmatrix} = \begin{pmatrix} a(r_{m1}) \\ a(x_{tg}) \\ \underline{g}(r, \theta_2) \\ a(x_{tg}) \\ \underline{g}(r, \theta_3) \\ a(x_{tg}) \\ a(r_{m2}) \\ a(x_{tg}) \end{pmatrix} \begin{pmatrix} a(r_{(01)}) \\ 80, 1) I_{ii} 0) \\ a(r_{m2}) \\ a(r_{m3}) \\ 0 (y 1) \\ a(r_{n(\pi)}) \\ \dots \end{pmatrix} \cdot \begin{pmatrix} [X_{tg} - X_i(\cdot)] \\ Y_{tg} - Y_i(\cdot) \\ (0) \end{pmatrix} \quad (3)$$

For the difference vector  $V_i$  the solution is:

$$V_i = (H^T H)^{-1} H^T VZ \quad (4)$$

With the calculated error the estimated target position  $tg^{(0)}$  can be corrected and the iteration can be restarted with this corrected position. As for the stopping criteria or convergence, the iteration step is defined as:

$$\frac{1}{2} ((x_{tg} - x_{tg}^{(0)})^2 + (y_{tg} - y_{tg}^{(0)})^2) < E \quad (5)$$

Based on the final estimated of the target's position, the relative angle between target and the car can be calculated easily by cosine law.

### 3. Phase error analysis

The noise term introduced in equation (1) is the main source which introduces phase errors at sensors. In our study we want to investigate the influence of this phase error on the angle estimation of the radar network. The targets were randomly generated in the simulation area as shown in Fig.2.

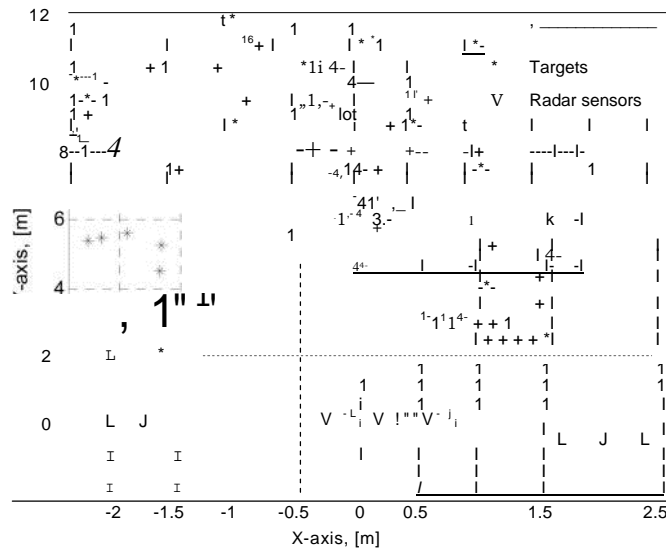


Fig. 2. Distribution of point targets across simulation area.

The source model was modeled as positive Gaussian and Uniformly distributed random variables with mean  $p$  and variances  $\sigma^2$ . The probability distribution functions are given by:

Gaussian:

$$\text{Uniform: } y=f(x|1,11,6)= \frac{1}{b-a} e^{-\frac{x-p}{\sigma^2}} \quad (6)$$

$$y=f(x|a,b)= \begin{cases} \frac{1}{b-a} & \text{for } x \in [a, b] \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

In our computer simulations we have simulated angle error under different kinds of noise environments. The variances of the distributions were changed from 1 cm to 10cm and we have assumed always positive errors. Fig.3 depicts the mean error in meters for position location of the targets on the area under different noise variances.

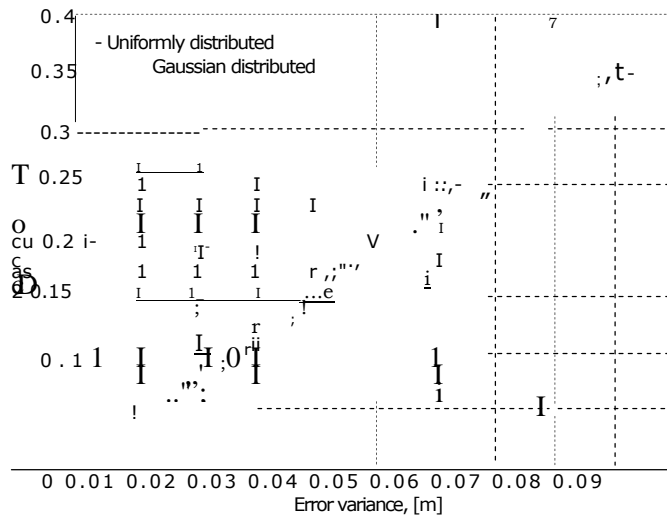
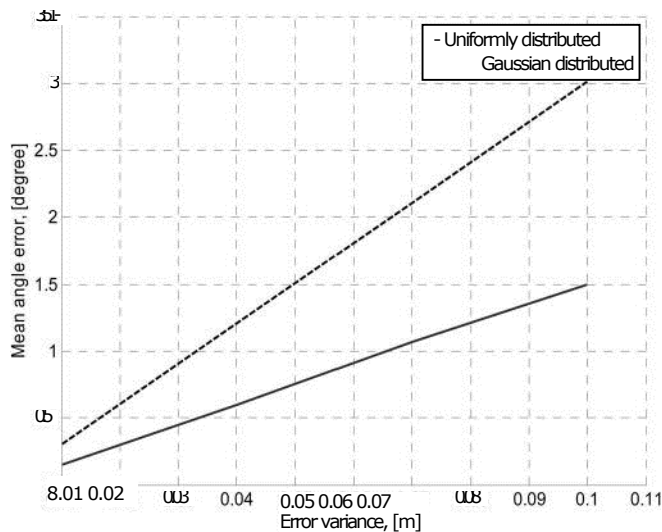


Fig. 3. Mean of target position location error under different noise distributions.

As it is seen from the Fig.3 there is a big difference between Gaussian and uniformly distributed noise effects on final target location. The angle between the target and the car estimation error yields similar relationship between two distributions. Fig.4 shows the mean of angle estimation error under different noise



distributions.

Fig. 4. Mean angle estimation error under different noise distributions.

As it was expected the Gaussian distribution introduces large errors due to positive assumption of errors. But the relationship will be saved even with positive and negative parts of Gaussian distributed errors, i.e. Gaussian distributed noise will have larger mean error.

From these results it is clear that there is linear relationship between noise variance and average error, even with small changes in noise variance there is a big changes in estimated position or angle of the target. When the radar system is being designed the sensitivity of the range measurement errors on angle estimation has to be taken into account and try to reduce those errors. When the noise behavior is described by uniform distribution the expected error is very small and can be tolerated, but when the noise is Gaussian distributed, then more attention is required in order to reduce angle estimation errors.

## 4. Conclusion

The effect of phase errors on estimated angle in an incident wave at radar sensors were analyzed by means of computer simulations. Based on computer we have simulated angle estimation under different noise distribution and variances. In case of Gaussian distributed noise errors the final position and angle estimates are largely affected by noise, in contrast, when the noise is uniformly distributed the position and angle estimates are less affected. Depending on noise environment radar sensors has to be designed carefully, especially under Gaussian distributed noises.

## 5. References

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