

# Numerical Experiments on Respiratory Motion Prediction Based on Locally Circular Motion Model

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**Abstract.** Motion-adaptive radiotherapy aims to deliver a conformal dose distribution to a tumor with minimal normal tissue exposure, by taking into account the tumor movement in real time. Motion of thoracic and abdominal tumors is induced by respiratory motion and it is required to compensate for system latency due to measurement and control. One of the most successful models for predicting respiratory motion is the locally circular motion (LCM) model. It characterizes the local respiratory behavior with a circular motion in an augmented plane and captures the natural evolution of respiratory motion. In this paper, we utilize the second-order extended Kalman filter based on LCM model and optimize filter parameters for each trace in an attempt to improve prediction accuracy. Numerical experiments are performed to evaluate and compare prediction accuracy of different prediction schemes.

**Keywords:** Radiotherapy, Respiratory motion, Prediction, Kalman filters.

## 1 Introduction

Motion of thoracic and abdominal tumors induced by respiratory motion often exceeds more than 1 cm which can compromise dose conformality significantly [1, 2]. Motion-adaptive radiotherapy aims to deliver a conformal dose distribution to a tumor with minimal normal tissue exposure, by compensating for the tumor motion in real time. There is system latency between acquisition of tumor position and repositioning of the treatment beam. This system latency can be up to several hundred milliseconds and it is required to predict the movement that has occurred during the system delay for the motion compensation. The problem of predicting respiratory motion has been intensively studied [2-6]. Nonparametric methods do not assume an explicit model on motion dynamics and learn the respiratory patterns to predict the future behavior from previous observations [3, 4]. The nonparametric inference models often require intensive training on a large data set. Parametric methods rely on a mathematical

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model that characterizes a local motion [5, 6]. The parametric approaches often have an advantage that can be implemented using efficient recursive prediction algorithms.

Recently, the use of MV treatment beam imaging combined with kV imaging has been investigated for real-time tracking of tumor position during the radiation delivery [7-10]. The use of the MV beam for acquisition of tumor position reduces the total imaging dose required for precise tracking. It provides geometric information of tumor to reduce kV imaging rate and dose. The advantages make the combined use of the MV and kV imagers a viable solution to the real-time tracking problem [9]. The sampling rates of the MV and kV imagers are maintained relatively low to comply with imaging system hardware and to limit kV imaging dose. For this case of low sampling rates where training data set is limited, parametric approaches have advantages over nonparametric methods. One of the most accurate parametric models for predicting respiratory motion is the locally circular motion (LCM) model proposed in [6]. The first-order extended Kalman filter (EKF) was implemented based on the LCM model and the prediction of the EKF was most accurate up to 0.5 sec prediction length region for 5 Hz sampling rate. In this paper, we utilize the second-order extended Kalman filter based on the LCM model and optimize filter parameters for each trace in an attempt to further improve prediction accuracy. Numerical experiments are performed to evaluate and compare prediction accuracy of different prediction schemes. Results of the experiments show that the first-order EKF is comparable with the second-order EKFs in prediction accuracy and the accuracy can be improved through use of parameters optimized for each trace.

## 2 LCM Model and Prediction of Respiratory Motion

The first and second-order EKFs are utilized to predict respiratory motion in order to compensate for system latency induced by measurement and control. The EKFs are implemented based on the LCM model, which characterizes the local respiratory behavior with a circular motion in an augmented plane [6]. Defining the discrete-time state vector by  $x(k) = [x(k) \ i(k) \ j(k) \ S2(k)]^T$ , we can write a discrete-time state equation to characterize the evolution of  $x(k)$  as

$$x(k+1) = f(x(k)) + v(k), \tag{1}$$

where

$$f(x(k)) = \begin{pmatrix} 1 - \sin\theta(k)T & \sin\theta(k)T & 0 \\ \sin\theta(k)T & 1 - \cos\theta(k)T & 0 \\ 0 & \cos\theta(k)T & 1 \\ 0 & 0 & 0 \end{pmatrix} x(k) + v(k). \tag{2}$$

The evolution of position  $x(k)$  is the projection of the planar circular motion onto the x-coordinate. The y-axis is an auxiliary axis augmented to define the circular motion.

We assume that the process noise vector  $v(k)$  is a zero-mean white sequence with its covariance matrix given by

$$Q(k) = E[v(k)v^T(k)] = \begin{bmatrix} q_1 T^3 & q_2 T^2 & 0 & 0 \\ 0 & q_2 T & 0 & 0 \\ 0 & 0 & q_3 T & 0 \\ 0 & 0 & 0 & q_3 T \end{bmatrix} \quad (3)$$

Here, the parameters  $q_1$ ,  $q_2$  and  $q_3$  are the power spectral densities of the continuous counterparts of the last three components of  $v(k)$ . These parameters each characterize the strength of possible changes in  $\dot{x}(k)$ ,  $\dot{y}(k)$  and  $\dot{z}(k)$  over the sampling interval

$T$ . The measurement equation of position  $x(k)$  is

$$z(k) = x(k) + w(k), \quad (4)$$

where  $z(k)$  is the observation at time  $k$  and  $w(k)$  denotes the corresponding additive measurement noise.

The goal is to predict the position at time  $kT + r$  ( $r > 0$ ), based on the measurement set of observations up to time  $k$ . Note that the state equation is nonlinear. The prediction of the future position  $x(kT + r)$  requires a nonlinear estimation. The first-order and second-order EKF is one of the simplest structures for implementing a nonlinear estimator. They rely on a first-order expansion and a second-order expansion of the nonlinear state dynamics  $f(x(k))$ , respectively, and calculate the

state estimate and its covariance matrix recursively. Due to page limit, details of the EKFs are not presented in this paper.

### 3 Numerical Experiments

The clinical data of 10 traces were used in our experiments. The data were obtained with the real position management system (RPM system, Varian Medical, Palo Alto, CA) by measuring position of fiducial markers placed on the patient's chest wall [4]. We adopt the normalized root mean squared error (nRMSE) as the performance measure for each trace defined by RMSE divided by the standard deviation of the sample values [4, 6]. The performance measure removes the adverse impact of the arbitrary scaling in RPM amplitude. Population nRMSE is also computed by averaging the trace-wise nRMSE over all traces.

The first and second-order EKFs were implemented based on the LCM model. Firstly, numerical experiments were performed to evaluate prediction accuracy of the first-order EKF (denoted LCM-1) and of the second-order EKF (denoted LCM-2) for 0.6 sec prediction length and 5 Hz sampling rate. The experimental results are listed in Table 1. The table presents the prediction accuracy of LCM-1 and LCM-2 in terms of nRMSE. It indicates that the improvement in accuracy of LCM-2 over LCM-1 is marginal. In the experiments the parameters,  $q_1$ ,  $q_2$  and  $q_3$ , of LCM-1 and LCM-2 were

**Table 1.** Comparison of prediction accuracy of LCM-1 and LCM-2 in terms of nRMSE (prediction length 0.6 sec, sampling rate 5 Hz).

Trace ID	1	2	3	4	5	6	7	8	9	10	Average
LCM-1	.44	.37	.51	.69	.50	.69	.65	.31	.48	.66	.543
LCM-2	.43	.37	.51	.69	.50	.68	.64	.31	.48	.65	.541
Error reduction (%)	0.2	0.3	0.2	0.3	0.4	0.7	0.9	0.0	0.6	0.3	0.37

**Table 2.** Comparison of prediction accuracy of LCM-1(opt) and LCM-2(opt) in terms of nRMSE (prediction length 0.6 sec, sampling rate 5 Hz).

Trace ID	1	2	3	4	5	6	7	8	9	10	Average
LCM-1(opt)	.42	.37	.47	.66	.49	.58	.60	.30	.48	.61	.509
LCM-2(opt)	.42	.36	.47	.65	.47	.57	.61	.30	.47	.62	.505
Error reduction (%)	0.0	2.4	0.2	0.5	4.5	1.5	-0.3	0.0	1.0	-0.5	0.78

set to 0.2,  $2 \cdot 10^{-4}$  and  $2 \cdot 10^{-3}$ , respectively. These parameters were optimized to minimize population nRMSE on a coarse grid in the parameter space.

Additional experiments were performed for the EKF with parameters optimized for each trace to minimize nRMSE. Let LCM-1(opt) and LCM-2(opt) denote the first and second-order EKFs with the optimized parameters, respectively. The experimental results are presented in Table 2. It shows that LCM-2(opt) does not improve prediction accuracy over LCM-1(opt). This implies that the uncertainty in the prediction cannot be characterized effectively with a second-order nonlinearity. The results indicate that the first-order EKF is the better scheme as a predictor, taking into account the computational cost.

Table 3 lists the above experimental results for LCM-1 and LCM-1(opt) for comparison. The error reduction of LCM-1(opt) over LCM-1 is approximately 6% in terms of the average nRMSE. The reduction, however, is not uniform over traces. It was null for trace 2 and 15% for trace 6. Although the error reduction depends on traces, the results suggest that the best predictor scheme is the first-order EKF with parameters optimized for each trace.

## 4 Conclusion

In this paper, the first and second-order EKFs were utilized to predict respiratory motion in order to compensate for system latency due to measurement and control. The EKFs were implemented based on the LCM model. The parameters of the first and second-order EKFs were optimized to minimize the average of normalized RMSE values over all traces. The parameters were also optimized for each trace to minimize the normalized RMSE. Results of numerical experiments show that the first-order EKF is comparable with the second-order EKFs in terms of prediction accuracy and the accuracy can be improved through use of parameters optimized for each trace.

**Table 3.** Comparison of prediction accuracy of LCM-1 and LCM-1(opt) in terms of nRMSE (prediction length 0.6 sec, sampling rate 5 Hz).

Trace ID	1	2	3	4	5	6	7	8	9	10	Average
LCM-1	.44	.37	.51	.69	.50	.69	.65	.31	.48	.66	.543
LCM-1(opt)	.42	.37	.47	.66	.49	.58	.60	.30	.48	.61	.509
Error reduction (%)	4.6	0.0	7.5	4.6	2.6	15.1	6.6	3.2	0.4	6.3	6.26

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