

Discrete Polar Complex Exponential Transform for Image Rotation Duplication Detection

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Abstract

With the aid of powerful photo editing software, region duplication is a easy way to create digital forgery images. Many prior approaches to detect and locate similar or identical regions are introduced in various application fields successfully. But in most case, these existing region duplication detection approaches can only detect some simple forensics which cloned region is moved to a different position in the same image, and are not effective when the duplicated regions have been subjected to geometrical distortions and added noise. In this paper, we describe a new region duplication detection approach that is robust to rotation of the duplicated regions. We propose an algorithm by constructing rotation moment invariants and extracting invariance features of images for Two-Dimensional (2-D) image forgery fields based on Discrete Polar Complex Exponential Transform (DPCET) which is one of three types of Discrete Polar Harmonic Transforms (DPHTs). This Discrete Polar Complex Exponential Transform approach is improved from traditional Polar Complex Exponential Transform (PCET) and this transform also encompasses the features of orthogonality and rotation invariance. We introduce a new 9×9 circle instead of a unit circle in DPCET to calculate features of image more precision. And various experiments were performed to prove that effectiveness of our approach in detecting duplicated forgeries, with translation, rotation, noise addition. And the experiment results also show that our approach is more robust to geometric distortions of the duplicated regions than other compared approaches.

Keywords: digital forgery images, duplication detection, rotation invariance, Discrete Polar Complex Exponential Transform, Discrete Polar Harmonic Transforms, 9×9 circle

1. Introduction

Over the past decade, information forensics and security become popular and attract much more people's attention. Image forensics is a key problem of information forensics and security field in today's society. Hence, image forgery detection is an important area of research in the field of social livelihood, scientific discovery, court evidence, financial document, news reports *et al.* With the development of the powerful photo processing software, some malicious users can perform forgery images to achieve their desired purposes. A common manipulation in tampering with digital images is known as region duplication, where a continuous portion of pixels are copied and pasted to a different location in the same image in order to conceal an important object in the scene [1]. Forgery meeting between President Clinton and Saddam Hussein is shown in Figure 1.

We focus on image forgery detection and aim at establishing correspondence between the same or similar duplicated parts which appear in one image or other different images.



Figure 1. Forgery Duplicated Image of Meeting between President Clinton and Saddam Hussein

Some general approaches are proposed to detect and locate duplicated forgery images. Fridrich [2] proposed approach base on Discrete Cosine Transform (DCT) coefficients which is one of the most widely-used techniques for duplicated forgery detection. But this approach has a low efficiency. In order to increase efficiency, Popescu *et al.* [3] proposed an approach based on the principal component analysis (PCA). PCA approach can reduce dimensionality of the matrix and get the aim to reduce computational complexity and cost. Cao *et al.* [4] and Huang *et al.* [5] also proposed an improved DCT mean coefficient approach and aimed to reduce the dimension of the feature vectors used by the Fridrich's algorithm. Zhang *et al.* [6] proposed an approach based on the pixel-matching for detecting and locating Copy-Move regions within an image. Chakraborty *et al.* [7] proposed an approach to joint histogram and mutual information for detecting illumination change in duplicated region. Kashyap *et al.* [8] proposed an approach based on wavelet decomposition.

The above detection approaches can detect forgeries in images well in the absence of post-processing operations. But this is not always the case. These general detection approaches typically do not provide valid evidence of region duplication or the location of the duplicated regions when the duplicated regions are suffered from geometric distortion. Considering image geometric distortion with translation, rotation and mirror is a key factor of image quality. Due to geometric invariant moments are highly concentrated on geometric transformation of image features, therefore, in the field of image duplication detection, geometric transformation (rotation, translation and scaling) invariance analysis is an important research content. In recently years, research on geometric distortion of forgery duplication image has been obtained fruitful achievements in the study of geometric moment invariants. Various kinds of moment invariants are proposed in the past decades. HU [9] moments are first proposed and Hu moments are widely-used techniques for duplicated forgery detection. Teh [10] ever proposed and evaluated a number of moments and addresses some fundamental questions. Latter various types of moment invariants are appeared to detect duplicated region of images. Popular and representative moment invariants approached are divided into several groups.

Scale Invariant Feature Transform (SIFT)

Amerini [11] proposed Scaling-Invariant-Features-Transform (SIFT) to reliably detect digital forgery images even if it is suffered from a large scaling transform. Pan *et al.* [1] proposed an approach by estimating the transform between matched SIFT feature key

points, which are insensitive to geometrical and illumination distortions, and then finds all pixels within the duplicated regions after discounting the estimated transforms. Sedaghat *et al.* [12] proposed a fully automated matching algorithm, called uniform robust SIFT, namely UR-SIFT. Chihaoui *et al.* [13] proposed to use the SIFT approach and match identical features by using the Singular Value Decomposition (SVD) approach to detect duplicated local characteristics or interest points of the images.

2) Zernike Moments (ZMs)

Ryu *et al.* [14] proposed Zernike moments which used as robust invariant features and then are able to detect accurately duplicated regions. When only rotation of the copied region takes place, the approach is superiority. Revaud *et al.* [15] proposed an accurate rotation-invariant similarity measure while retrieving the optimal in-plane rotation angle. Chen *et al.* [16] proposed an approach to derive a set of combined geometric-blur invariants based upon orthogonal Zernike moments (OZMs). LI *et al.* [17] propose combining both the magnitude and phase coefficients to form a new shape descriptor, referred to as invariant ZM descriptor (IZMD).

3) Fourier Mellin Moments (FMMs)

Bayram *et al.* [18] proposed to extract image characteristics from blocks of image by applying the Fourier Mellin Transform (FMT). Sheng *et al.* [19] proposed Orthogonal Fourier-Mellin moments (OFMMs) to describe rotation invariant patterns. Singh [20] propose a new approach to construct rotation invariant which combines the merits of the Fourier-Mellin-Transform (FMT) and Modified Phase-Only Correlation (MPOC) techniques. The experiment results show that this approach has a high resolution in partial palm print images detection which is rotated and added Gaussian noise.

4) Polar Harmonic Transforms (PHTs)

Yap [21] introduced a set of 2D of transforms, namely Polar Harmonic Transforms (PHTs) in 2010. PHTs are a set of 2D transforms defined over the unit circle in the polar coordinate system which can be used to generate rotation-invariant features. Li *et al.* [22] used PHTs to extract the features of the circular blocks, which are then used to perform block matching and locate duplicated region of the forgery image. PHTs [21-23] encompass the orthogonality and invariance which is superiority to ZMs, PZMs and SIFT. Compared ZMs, PZMs, SIFT and OFMM, computation of PHTs kernel is simple and stable [21]. Hence PHTs have provided a powerful tool for image detection.

In this paper, we do research on image forgery field, mainly focus on the duplicated region detection. The essential challenge in duplicated region detection field is how to detect, recognize and locate one object in an image location. We propose a new Discrete Polar Complex Exponential Transform which is belong to Discrete Polar Harmonic Transforms approach to construct translation, rotation moment invariants and extract translation, rotation invariance to detect duplicated region, even if these region are subjected to geometric distortion, such as translation, a large degree rotation tampered, and additive noise. We introduce a new 9×9 circle instead of a unit circle in DPCET to calculate features of image more precision.

The rest of the paper is organized as follow: in Section 2, we review the Two-Dimension Polar Harmonic Transforms. In Section 3, we propose our approach and define Discrete Polar Harmonic Transforms (DPHTs), mainly DPCET. Detail to discuss DPCET to construct rotation moment invariants and extract region invariance features. In Section 4 we describe a few experiments which results we have performed to evaluate the performance of the proposed approach. We further demonstrate its practical performance by introducing Image False Detecting Rate (FDR) and comparing with other approached in detecting precision rate. Finally, the conclusions are given in Section 5.

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2. The Review of the 2-D Polar Harmonic Transforms

In this section, we have grouped three types' transforms which is Polar Complex Exponential Transform (PCET), Polar Cosine Transform (PCT), and Polar Sine Transform (PST) under the name Polar Harmonic Transforms as the kernels of these transforms. These three type transforms are harmonic basic waves of PHTs.

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$$f^2$$

(9)

i
□

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$$\frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp(-j2\pi(xu + yv)) dx dy$$

2.1 Definition of Polar Complex Exponential Transform (PCET)

The general 2-D moment definition using a moment weighting kernel, namely basis function $\phi(x, y)$, and an image intensity function $f(x, y)$ is given as

(1)
 Given a 2D image function $f(x, y)$, it can be transformed to it from Cartesian coordinate to Polar coordinate. The transform in this set of function is defined as [21, 22]

$$\tan^{-1}(\frac{v}{u})$$

$$M_c = \int_0^{2\pi} \int_0^{\infty} f(r, \theta) \exp(-j2\pi(r^n \cos(n\theta)u + r^n \sin(n\theta)v)) r^{n-1} dr d\theta$$

(2)

Where M_c is Polar Complex Exponential Transform (PCET) of $f(x, y)$, with order n and repetition l. where $\phi(x, y)$ is a basis function of PCET,

indicates the complex conjugate of the . For PCET, is consisted of radial and circular components and can be defined as

$$H_{nlr} = \int_0^{2\pi} \int_0^1 H_{nlr}^* r dr d\theta$$

1

(3)

Where is the radial kernel, i is a imaginary and , r indicates radius defined on unit circle and indicates rotation degree which are defined as

27

,

x

(4)

1

is satisfy orthogonal condition, and defined as [21]

(5)

27

$$M_C = Q_j^{-1} [H^S]$$

2.2 Definition of Polar Cosine Transform (PCT) and Polar Sine Transform (PST)⁽⁶⁾

Similar to the form of PCET, Polar Cosine Transform (PCT) is defined as [21]

2

a)

*

(7)

Polar Sine Transform (PST) is defined as H^S

$$H^S = \int_0^{2\pi} \int_0^1 H^S \sin(\theta) r dr d\theta$$

sin(

(8)

Where the basis function of PST and PCT are defined as

Where $O_n = \begin{cases} 1 & \text{if } n = 0 \\ 2 & \text{if } n \neq 0 \end{cases}$,

3. Proposed Approach based on Discrete Polar Complex Exponential Transform

θ

3.1 Constructing Rotation Moment Invariant

From (2), we supposed $\theta = \theta_0 + \Delta\theta$ and get derivation $d\theta = \Delta\theta$. We also supposed $e^{i\theta} = f(r, \theta)$, Where f is rotated a degree. Hence, (2) can be rewritten as

$$f(r, \theta) = \sum_{n=-\infty}^{\infty} \int_0^R f(r, \theta) e^{in\theta} r dr \quad (10)$$

(10) is also rewritten as $\int_0^R f(r, \theta) e^{in\theta} r dr$ (11)

$$\int_0^R f(r, \theta) e^{in\theta} r dr$$

$$(\int_0^R f(r, \theta) e^{in\theta} r dr)$$

From (10), (11) we can construct a rotation moment invariant as

$$\int_0^R f(r, \theta) e^{in\theta} r dr$$

(12)

3.2 Definition of Discrete Polar Complex Exponential Transform (DPCET) and Rotation Invariance Extraction

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Given a 2D signal function $H_{nl}(x, y)$, the Polar Complex Exponential Transform is defined on a unit circle in Cartesian coordinate as [21].

$$M_{nl} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H_{nl}(x, y) e^{-j2\pi(xu + yv)} dx dy \quad (13)$$

Where

$$\text{and } H_{nl}(x, y) = H_{nl}(r \cos \theta, r \sin \theta) = H_{nl}(r, \theta)$$

The Polar Complex Exponential Transform function defined on a unit circle is shown in Figure 2.

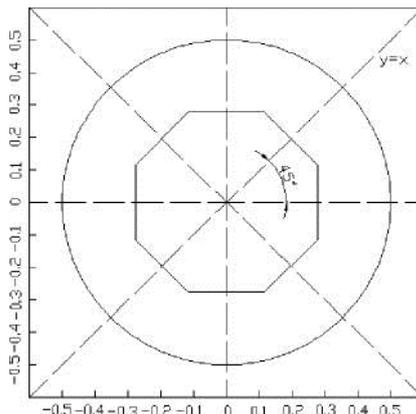


Figure 2. The Polar Complex Exponential Transform Function Defined into a Unit Circle

The above discussion has been only limited to apply in continuous image domain. In practice, this is not always a case. Images which we obtain are defined on a discrete domain. Given an image with size of $M \times N$, (14) shows the image is necessary for proper normalization. Hence, its pixels are mapped on discrete unit circle domain. Supposed x and y are discrete coordinates.

$$f(x, y) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} f(p, q) e^{-j2\pi (px + qy)} \quad (14)$$

Hence, discrete of (10) can be defined as

$$(15)$$

Supposed $H(x, y)$, (13) and (15) can also be written as

$$M \times N \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} [H(x, y)] f(x, y) \quad (16)$$

In our detection technique, we proposed a detected circular template which size is 9×9 to detect the image. Figure 3 shows as

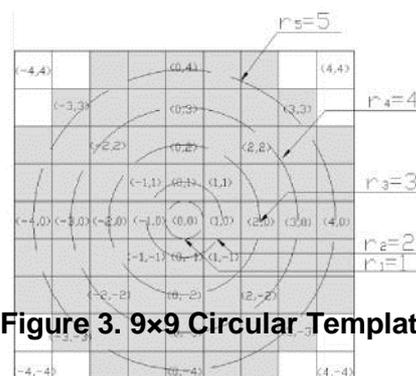


Figure 3. 9x9 Circular Template

The template is a circular block which is consisted of gray grid pixels. From Figure 3, (16) can be rewritten as

$$M = \sum_{x=-4}^4 \sum_{y=-4}^4 [f(x, y)] * [g(x, y)] \quad (17)$$

tampered experiments which mainly are duplicated region rotated from 0° to 180° and added different Gaussian noise. The Fig.5-12 are performed to gauge the performance of DPCET. And then we input 100 nature images which are not tampered and use DPCET approach to detect these not being tampered images and obtain false detecting rate of these images. In the last experiment, we compare our performances with Ryu's [14] and Singh's [20]. The experimental results are shown that our approach is relatively more accuracy than compared approaches. In our experiments, MATLAB 7.0 Math Software is applied to analysis the approach in our paper and other relevant approaches.

4.1 Detection Precision of DPCET

We download a variety of 100 nature images from the Internet for our experiments. We take these 100 test images with a set of forgery ways by Photoshop. We tempered the image region with the ways of translation, clockwise rotation, and additive Gaussian noise. To further evaluate the performance of our approach in rotation duplicated image, we duplicate region and rotate it from 0° to 180° . Then we get these duplicated rotation sample and test them which degrees are separated by 5° . Further more, the duplicated region is subdivided into three type sizes which is size of 32×32 , size of 64×64 and size of 128×128 . The above tampered image samples are tested with the proposed approach in this paper, and the tested samples and results are showed in Figure 5-Figure 12:



Figure 5. Origin Image of Panda-cake

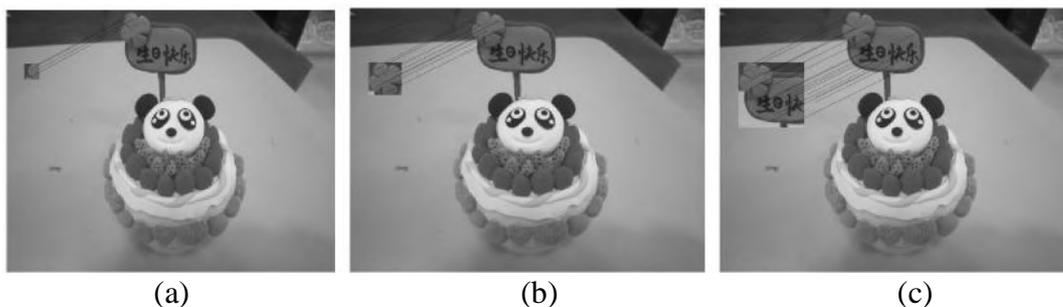


Figure 6. Detection for translation tampered (0° clockwise rotation) and some matched pixels are connected them with lines. (a) Duplicated region size of 32×32 pixels; (b) Duplicated region size of 64×64 pixels; (c) Duplicated region size of 128×128 pixels.

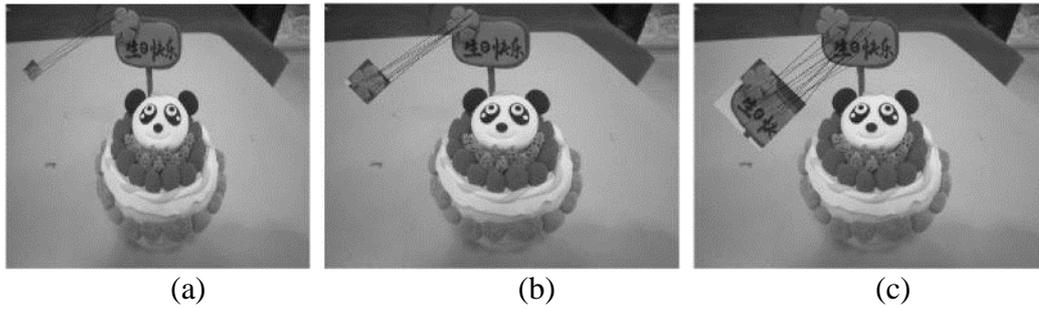


Figure 7. Detection for 45° clockwise rotation tampered and some matched pixels are connected them with lines. (a) Duplicated region size of 32×32 pixels; (b) Duplicated region size of 64×64 pixels; (c) Duplicated region size of 128×128 pixels.

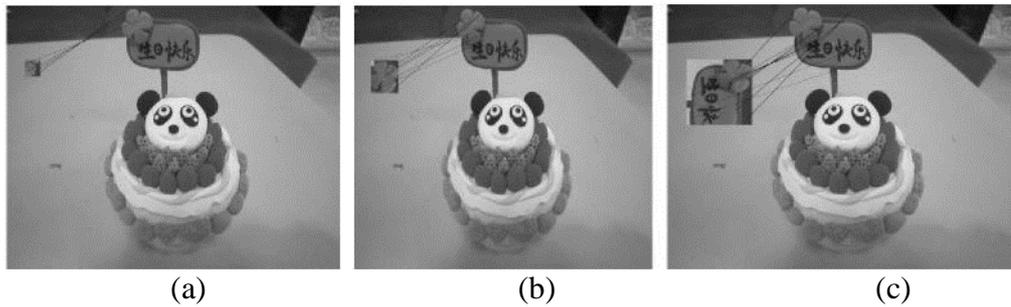


Figure 8. Detection for 90° clockwise rotation tampered and some matched pixels are connected them with lines. (a) Duplicated region size of 32×32 pixels; (b) Duplicated region size of 64×64 pixels; (c) Duplicated region size of 128×128 pixels.

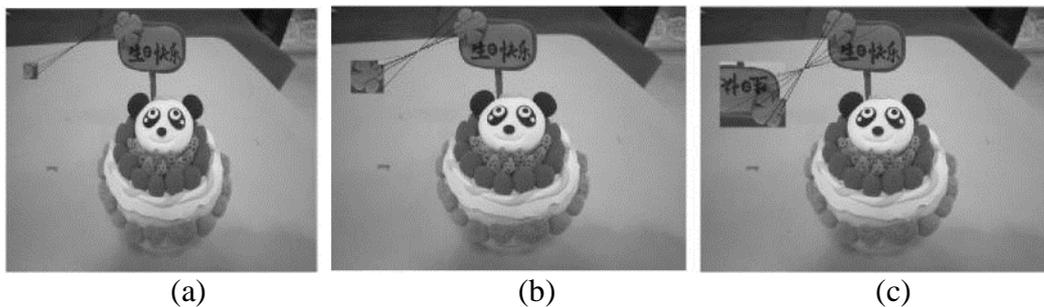


Figure 9. Detection for 180° clockwise rotation tampered and some matched pixels are connected them with lines. (a) Duplicated region size of 32×32 pixels; (b) Duplicated region size of 64×64 pixels; (c) Duplicated region size of 128×128 pixels.

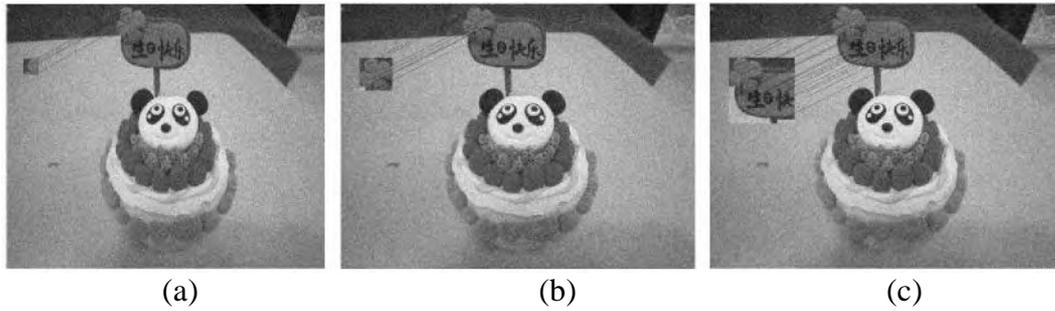


Figure 10. Detection for translation, additive Gaussian noise () tampered and some matched pixels are connected them with lines. (a) Duplicated region size of 32×32 pixels; (b) Duplicated region size of 64×64 pixels; (c) Duplicated region size of 128×128 pixels.

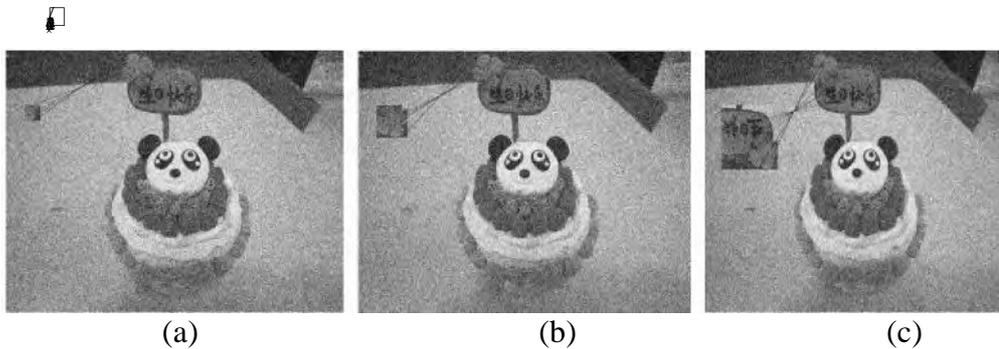


Figure 11. Detection for 180° clockwise rotation, additive Gaussian noise () tampered and some matched pixels are connected them with lines. (a) Duplicated region size of 32×32 pixels; (b) Duplicated region size of 64×64 pixels; (c) Duplicated region size of 128×128 pixels.

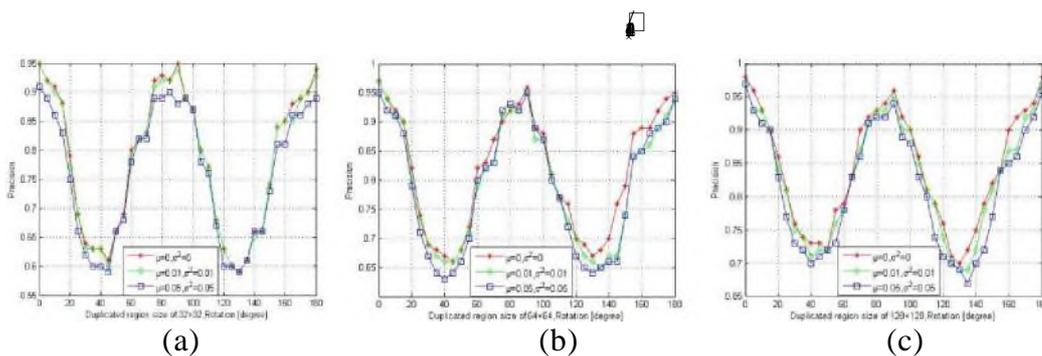


Figure 12. Detection precision rate of duplicated and tampered region with additive Gaussian noise, and rotated from 0° ~ 180° degrees (separated by 5°). (a) Duplicated region size of 32×32 pixels; (b) Duplicated region size of 64×64 pixels; (c) Duplicated region size of 128×128 pixels.

From above experimental results, our DPCET approach in detecting duplicated images which are suffered from translation and rotation will still keep certain precision rates, especially, when the rotated degree is close to 0° and 180°. But when the rotated degree is close to 90° and 270°, the relative detection precision rate is dramatically declined. And Additive Gaussian noise is not apparently acting on our image detection. This is because the duplicated region which is rotated close to 90° and 270° led to the change the

relatively position of neighboring pixels which will be filled a certain pixel values. And the duplicated region which is rotated close to is detected more easily and

precision. When the duplicated region sizes is lager, such as 128×128, is located, detected more easily and has higher precision compared with other smaller sizes.

4.2 Image False Detecting Rate of DPHTs in Our Approach

From section 4.1, we can prove that our approach precision is well. But this result is established on a supposed fact that an image is tampered and exists duplicated region. In practice, this is not real case. It is usually not prior to know whether the tested image is tampered. For an nature or not being tampered image, a true case is our method does not detect any continuous duplicated region. In paper [1], region based performance measures such as pixel detection accuracy (PDA), pixel false positive (PFP) rates are useful. In this paper, in order to further demonstrate the benefit of DPHTs approach, we present image False Detecting Rate (FDR). We also input 100 nature images which are not tampered. We use DPHTs to detect these not being tampered images and obtain the image false detecting rate (FDR). The image false detecting rate is shown in Table 1.

Table 1. Average Image False Detecting Rate (Percent) for our Approach on 100 Not Being Duplicated Images under Certain Angles and Additive Gaussian noise

Size	32×32	64×64	128×128
Not being tampered	2%	0%	0%
Additive noise	3%	0%	0%
Additive noise	2%	1%	0%
Additive noise	3%	1%	0%

Table 1 shows the image false detecting rate of our approach which is quiet well. And it refers to the size of the duplicated region which we supposed is not smaller than $M \times N$ () pixels. Judged threshold of matching pixels is not lower than $70\% \times M \times N$. Other wise, it is not a true forgery image. Figure 13 shows the matching pixels which connected them with lines in an image.



Figure 13. Matching Pixels in an Image

4.3 Comparison for Copy–Move Authentication Approaches

We also implemented the approaches with Ryu's [14] approach and Singh's [20] approach. We compared detection rate in duplicated rotation region of these approaches in Figure 14.

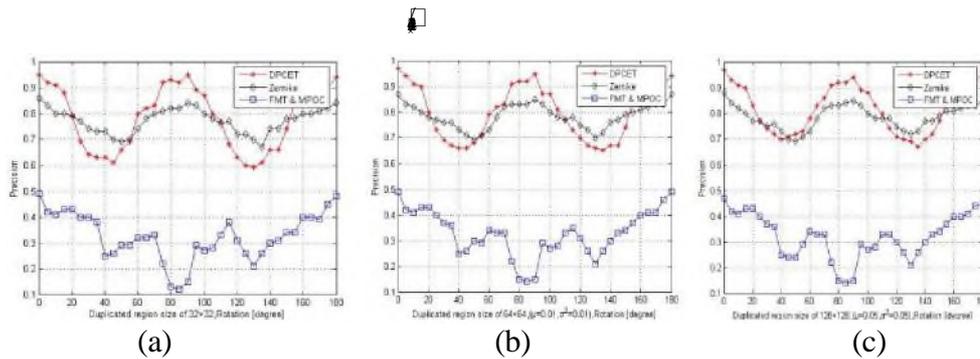


Figure 14. Detection precision rate of our DPCET approach and Zernike approach in [14], FMT & MPOC approach in [20] with additive Gaussian noise from 0° ~ 180° rotated (separated by 5°). (a) Duplicated region size of 32x32 pixels; (b) Duplicated region size of 64x64 pixels; (c) Duplicated region size of 128x128 pixels.

Figure 14 shows our proposed approach is higher precision than FMT & MPOC approach in [20], and in most case, average precision rate of our approach is also higher than Zernike approach in [14].

5. Conclusions

With the rapid development of computer technology, normal users can perform duplicated region operation easier for creating a realistic composite image. As such it is also significant to detect these duplicated operations for verifying and authenticating the originality of the digital images. In this paper, we have presented the overview of the advance techniques in image forensics and security applications. We have focused on the localization and detection of image duplicated forgery, for which we advocate Discrete Polar Harmonic Transforms as geometry feature representation of small duplicated overlapping image blocks which size is at least 32x32 pixels. For the geometric features, we proposed to construct rotation moment invariant and extract its rotation invariance. The

detail formula derivation is discussed and analyzed in section 3. And the latter experiments can all be viewed in support of the above decision of formula derivation. All experimental results strongly suggest that localization and detection performance greatly depends on image geometric features, such as translation, rotation. The experiments in section 4.1 shows that the proposed approach can detect duplicated forgery images successfully, and furthermore, even if the duplicated forged image is subjected to rotation transform and additive Gaussian noise. When the size of duplicated region is larger than 32×32 , we will achieve higher level of accuracy. The experiments in section 4.2, we introduce the image False Detecting Rate (FDR) to authenticate misjudgment of our proposed approach in detecting nature or no being tampered image. In section 4.3, we compared the robustness of our approach with the previously proposed approaches which also research on duplicated rotation region in paper [14, [20]. Our proposed approach based on rotation moment invariants show more excellent. But the precision of our proposed approach in detecting of is not relatively high. This is due when the duplicated region or sub-image which is rotated to closely will led to change the relatively position of neighboring pixels which are realize to fill a certain pixel values greatly. In addition, we slide one pixel each time to detect the whole image which will lead to low efficiency. So in the next step, we will accelerate correlated work and improved detection approach to enhance robustness, increase precision and improve efficiency, make the approach for the duplicated rotation region of image detection more efficient.

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