

# Localization of a Self-Driving Vehicle by Extended Kalman Filtering with Fixed Gains

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**Abstract.** In this paper, a sensor fusion method for self-driving vehicles is introduced. The proposed sensor system consists of a GPS, an IMU with three-axes accelerometers, three-axes gyroscopes, and three-axes magnetometers, an encoder at wheels, and a potentiometer that measures the steering angle. For the complete estimation of the absolute position and orientation, the proposed method estimates the driving velocity using a kinematic Kalman filter, and then the orientation angle and absolute position are calculated by the recursive least squares. The proposed method is verified by experimental results in this paper.

**Keywords:** Sensor fusion, Kalman filter, Random process

## 1 Introduction

In the field of automotive engineering, many researchers are focusing on the development of a self-driving technologies. Many vehicle manufacturers such as BMW have actively investigated the self-driving technology and started implementing it into commercialized vehicles[1]. For examples, a self-parking system automatically controls the driving and steering actuators for safe parking[2], and an automatic braking system automatically reacts to an emergency situation[3]. Various key technologies are integrated into the self-driving system, e.g., driving motor control, real-time path planning, and so on. For the precise and safe control of the vehicle's location, the localization and motion control of the vehicle are of important issues. Most of the localization systems rely on a global positioning system (GPS). The GPS, however, is subjected to an error due to climate conditions, e.g., cloud coverage and humidity, and structural obstacles, e.g., buildings and mountains. An inertial measurement unit (IMU) can be used to compensate the GPS. The IMU contains accelerometers, magnetometers and gyroscopes to measure the object's acceleration and tilt. The IMU, however, is also subjected to a drift problem due to integration. Therefore the vehicle's location should be calculated using a combination of the multiple sensors.

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This paper verifies the sensor fusion algorithm presented in [4] with experimental result. Driving an electric cart on a road, and obtaining the data from IMU, encoder, potentiometer and GPS, the estimate of the absolute position by the proposed sensor fusion algorithm is verified by comparing the estimate of the absolute position with the true absolute position.

## 2 Kinematic Analysis of a Vehicle

In general, the absolute position and orientation of an object in the three-dimensional space has six degrees of freedom (DOFs) due to the three-dimensional translational movements and three-dimensional rotations. The vehicles, however, can be analyzed by three DOFs, because their motion is constrained on a plane (i.e., the road). Figure 1a shows the kinematics of a vehicle on a plane. In the figure,  $l_{vehicle}$  represents the distance between the front wheels and rear wheels,  $v(k)$  is the travel velocity,  $a_y(k)$  is the frontal acceleration that is perpendicular to the shaft of real wheels,  $\rho(k)$  is the radius of curvature, and  $\theta(k)$  represents the steering angle. It is assumed that the tire friction is large enough such that the wheels are not slipped in any case.

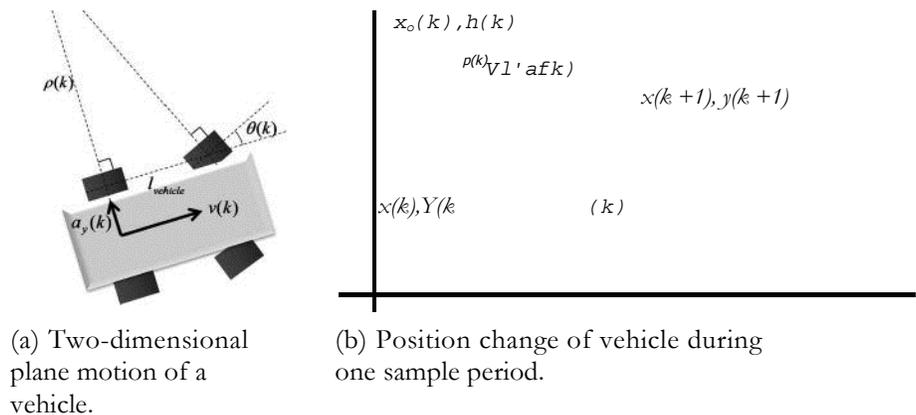


Fig. 1: Kinematic analysis of a vehicle.

Let the absolute position (i.e., the latitude and longitude) and orientation of a vehicle are  $x, y$ , and  $\theta$ , respectively. The orientation is defined as the angle of the vehicle with respect to the absolute coordinate system. If the steering angle,  $\theta(k)$ , is not zero, as shown in Fig. 1a, the vehicle follows a circular trajectory, where the radius of curvature is

$$\frac{l_{vehicle}}{\tan \theta(k)} \quad (1)$$

In the proposed method, the absolute position and orientation (i.e.,  $x$ ,  $y$ , and  $\theta$ ) are to be updated (i.e., estimated) every sampling instance. Figure 1b shows a case where the vehicle follows a circular trajectory with the constant steering angle and velocity. Given the current position and orientation (i.e.,  $x(k)$ ,  $y(k)$ , and  $\theta(k)$ ) the radius of curvature can also be defined as

$$r(k) = \frac{v(k)T}{a(k)} \quad (2)$$

where  $v(k)$  is the travel velocity of the vehicle, and  $T$  is the sampling period.  $a(k)$  is the change of the orientations at the  $k$ th and  $(k + 1)$ th locations. Namely,  $a(k)$  is the difference between  $\theta(k + 1)$  and  $\theta(k)$  is, i.e.

$$\theta(k + 1) = \theta(k) + a(k) \quad (3)$$

Substitution of Eqs. (1) and (2) into (3) yields

$$\theta(k + 1) = \theta(k) + \frac{v(k) \tan \theta(k)}{r(k)} \quad (4)$$

Notice that the orientation can be updated by the current speed and the steering angle, as in Eq. (4).

From the geometrical representation shown in Fig. 1b, the absolute position at  $(k + 1)$ th step can be defined as

$$p(k + 1) = p(k) + R(k) \begin{bmatrix} \sin \theta(k) \\ \cos \theta(k) \end{bmatrix} \cdot \quad (5)$$

where

$$p(k) = \begin{bmatrix} x(k) \\ y(k) \end{bmatrix} \quad \text{and} \quad R(k) = \begin{bmatrix} r(k) \cos \theta(k) \\ r(k) \sin \theta(k) \end{bmatrix} \cdot \quad (6)$$

and  $a(k)$  and  $\theta(k)$  are as defined in Eqs. (3) and (4). The state space equations in Eqs. (4) and (5) are the foundations of the proposed localization algorithm.

### 3 Experimental Verification

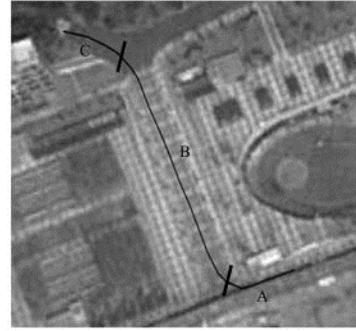
In this section, we verify the proposed sensor fusion algorithm proposed in [4] by experimental result.

The experiment model of the vehicle is shown in Fig. 2a. The experiment model is an electric cart powered by battery. The driving mode of the cart consists of an automatic mode and a manual mode. In the manual mode, the vehicle can be driven by the steering wheel, accelerator pedal and brake, and in the automatic mode, the vehicle can be controlled with LabVIEW and FPGA boards. The distance between front wheels and rear wheels,  $G_c$  is

$G_c$  is 2.15m and the radius of wheel,  $R$ , is 0.25m. An encoder is installed on the rear wheel, and a potentiometer is installed on shaft of the steering wheel. An IMU is placed



(a) Experiment model.

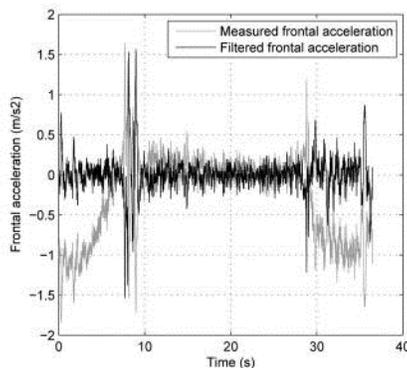


(b) Trajectory.

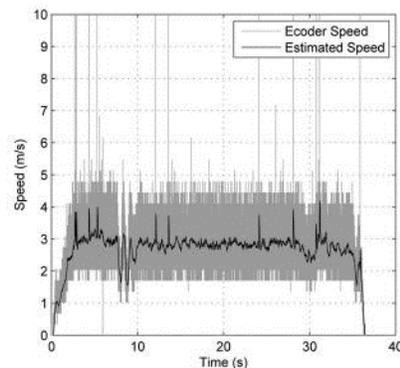
Fig. 2: Experiment environments.

on the front part of the vehicle, and GPS sensors are on the four edges of the ceiling. Figure 2b shows a true trajectory in the experiment. Upper side of Fig. 2b is North side. In the experiment, initial absolute position is the latitude of  $37^{\circ}33'2.66'' N$ , and the longitude of  $126^{\circ}56'35.27'' E$ . The sections labeled as A, B, and C in the Fig. 2b have the slopes of 20, 5, and 10 degrees, respectively. High rise buildings and tall trees do not exist around the experiment trajectory.

### 3.1 Estimated Speed and Orientation



(a) Frontal acceleration.



(b) Estimated of the speed.

Fig. 3: Frontal acceleration and speed.

The experiment trajectory has the slope of the road, and thus the gravity affects to frontal acceleration, which causes an error in the estimation of the speed and travel distance. Therefore, a measured frontal acceleration may be different from the true frontal acceleration. When the slope of the road increases,

the measured frontal acceleration becomes negative, as shown in Fig. 3a. The proposed sensor fusion algorithm, however, does not consider the slope of the road, and thus additional filter is needed for accurate localization. Figure 3b shows the estimated speed of the cart. The estimated speed by the encoder shows the noise of  $\pm 1\text{m/s}$ . On the other hand, the estimated speed by Kalman filter is free from the noise, and it is close to the mean of the estimated speed by the encoder. Therefore, the estimated speed by Kalman filter can be regarded as the true speed of the cart.

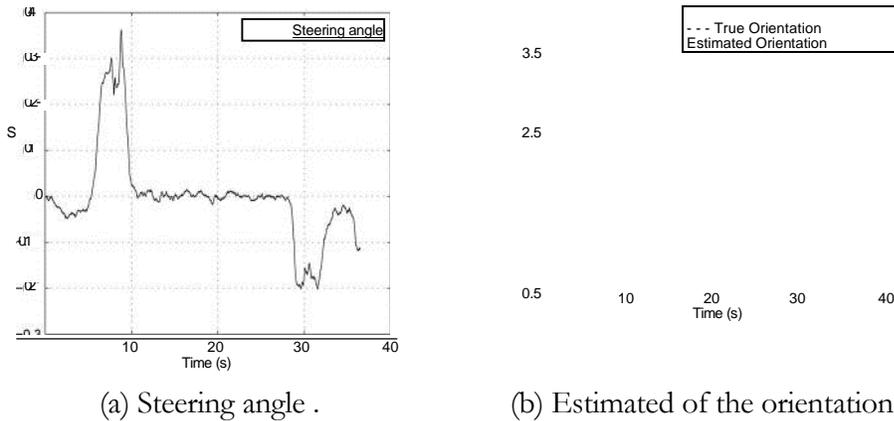
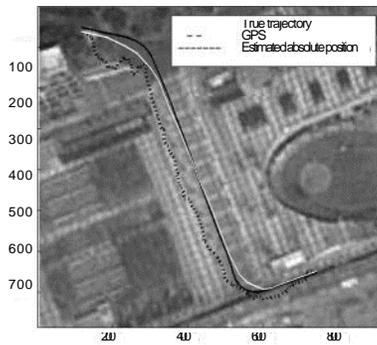


Fig. 4: Steering angle and orientation.

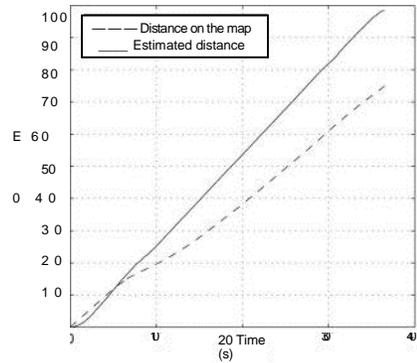
Figure 4a shows the steering angle of the cart. The steering angle is positive when the front wheels turn to the right hand side. Figure 4b shows the orientation of the cart. The orientation is the angle of the central axis of the cart with respect to absolute longitudinal axis. In Fig. 2b, initial direction of the cart is a south-west, which is about 3.5 radian from longitudinal axis as shown in Fig. 4b. Therefore, the result of Fig. 4b is similar to Figs. 2b and 4a. Also, the true orientation and the estimated orientation show the same trend.

### 3.2 Estimated Absolute Position

Figure 5a shows the absolute position of the cart. Although the estimated absolute position by the proposed sensor fusion algorithm shows a discrepancy from the true trajectory the magnitude of the error is small. This error is because of the GPS sensor and the slope of the road. The accuracy of the GPS sensor used in the experiment was remarkably low, which deteriorated the accuracy of the estimated absolute position significantly. Also, the update gain of the sensor fusion algorithm is in inverse proportion to the accuracy of the GPS. Another reason of the error is the slope of the road. Since the length on the map shown in Fig. 2b is one projected onto the two-dimensional plane, which does not consider



(a) Vehicle trajectory.



(b) Driving distance.

Fig. 5: Vehicle trajectory and driving distance.

the slope of the road, the estimated distance and the distance on the map show a discrepancy.

## 4 Summary

This paper, verifies the sensor fusion algorithm in [4] by experimental result. The estimated absolute position was similar to the true absolute position. The proposed sensor fusion algorithm is expected to be applicable to self-driving vehicles.

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