Time-Varying Extended Kalman Filtering for Localization of a Self-Driving Vehicle

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Abstract. In this paper, a sensor fusion algorithm based on the extended Kalman filtering for the localization of self-driving vehicles is introduced. The proposed sensor system consists of a GPS, an IMU with three-axes accelerometers, three-axes gyroscopes, and three-axes magnetometers, an encoder at wheels, and a potentiometer that measures the steering angle. For the complete estimation of the absolute position and orientation, the proposed method estimates the speed using a kinematic Kalman filter, and then the orientation angle and absolute position are calculated by the recursive least squares. The proposed method is verified by simulation results in this paper.

Keywords: Sensor fusion, Kalman filter, Random process

1 Introduction

In past few decades, vehicle technology has been dramatically improved for better power-efficiency and driving comfort. For examples, the hybrid engine technology has provided a solution to the limitations of gasoline engines[1], and active suspension systems have improved the driving comfort by controlling the damping coefficient between the body of vehicle and tires[2]. In recent years, many researchers have focused on the intelligence of vehicle which enhances the drivability and safety of the vehicle[3]. Such intelligence technology allows the vehicles to monitor the status of a driver and to react to an emergency situation automatically. For example, a blind driver drove a car with the assistance of the self-driving technology by Google[4].

For precise control of self-driving vehicles, it is important to measure the absolute position and orientation of the vehicles in real-time. In general, path planning algorithms of self-driving vehicles make decisions based on the vehicle's current and target positions. Therefore, the accuracy of the vehicle's absolute position measurement determines the performance of self-driving vehicles. Unfortunately, however, such an integrative sensor system that is able to measure the absolute position and orientation of a vehicle does not exist in practice. A

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global positioning system (GPS) estimates the absolute position of a vehicle from satellite signals, but its reliability and accuracy are not acceptable for the precise control. An inertial measurement unit (IMU) is often used to complement the GPS, but it is always subjected to a drift problem due to integration. The driving distance may be measured by an encoder connected to the wheels, but a reference point must be known prior to the experiment in order to convert the rotation angle of the wheel into the absolute position of the vehicle.

In order to obtain a better estimation by taking advantages of these sensing methods, various sensor fusion algorithms have been proposed; e.g., a complementary filtering of the GPS with an inertial measurement unit (IMU)[5]. The IMU is a sensor system that includes a three-axis accelerometer, a three-axis gyroscope, and a three-axis magnetometer. Since the position estimation by a GPS and one by an IMU show different characteristics in the frequency domain, they can be selectively combined in the frequency domain by the complementary filtering. This method, however, does not yet provide full information regarding the kinematics and dynamics of a self-driving vehicle, which are necessary for precise motion control.

In this paper, an advanced sensor fusion algorithm is proposed for the estimation of the absolute position (i.e., the latitude and longitude), speed, and orientation of a self-driving vehicle. The proposed method utilizes an encoder installed at wheels and a potentiometer measuring the steering angle, as well as a GPS and an IMU. Mathematical relationships between the sensor signals are analyzed based on the kinematics of the vehicle. Following the concept of the extended Kalman filter for nonlinear systems, the sensor signals are combined in real-time in an optimal manner. The proposed method is robust to the noise in measurements.

2 Estimation of Absolute Position and Orientation 2.1

Vehicle Speed Estimation

An encoder is a sensor that measures the rotation of a wheel with a counter. Therefore, the travel distance, $s(t)$, can be calculated from the encoder measurement. On the other hand, the frontal acceleration, $a(t)$, which is the second derivative of the travel distance, can be measured by an inertial measurement unit, IMU. Therefore, in an ideal case $s(t)$ and $a(t)$ are related by

$$s(t) = \iint a(t)dt^2 + v(0)t + s(0) . \qquad (1)$$

where s(0) and v(0) the initial conditions. Since a vehicle starts driving from zero speed and the initial travel distance can be set to zero, it is reasonable to assume that s(0) = 0 and v(0) = 0. With these conditions, Eq. (1) can be expressed as a state space equation in the discrete-time domain, i.e.

$$x(k+1) = Ax(k) + Ba(k) .$$
$$s(k) = Cx(k) . \qquad (2)$$

where

$$A = 11 \quad [0\ 1]\ ' \qquad B = [TT21 \qquad C = [1 \qquad , x(k) = [^{s(k)}1 \quad Lv(k)] \qquad (3)$$

In practice, an encoder is subjected to an error due to the unknown deformation of tires and the slip of wheels. Moreover, the error is accumulated as driving distance increases. Also, in the case of an IMU, an accumulated error by integration is unavoidable. In this paper, a kinematic Kalman filter is utilized to reduce the magnitude of the error in the estimation of the travel distance, *s(t),* and speed, v(t), by fusion of the two different sensors. Suppose that the measurements from the encoder and IMU are

$$a(k) = aT (k) + EAcc(k) \qquad (4)$$

$$s (k) = s_T (k) + E_{En,c}(k) \qquad (5)$$

where $a_T (k)$ is the true frontal acceleration and $s_T (k)$ is the true driving distance. $EA_{cc}(k)$ and $EE_{nc}(k)$ are zero-mean white Gaussian noises with the covariances of $AA_{„}$ and AE_{ric}, respectively. All the noises are assumed to be uncorrelated. The noise in the encoder signal, *EE,,,(k),* may show a drift due to the unexpected tire deformation in practice, but the drift is neglected for the sake of simplicity. With this setting, a Kalman filter problem can be formulated as follows.

$$ic(k + ilk) = Alc(klk) + B\ a(k) \qquad (6)$$

$$"X(k +11k +1) = \qquad + ilk)\ F\ [s(k + 1) — Ccc(k \pm 11k)]\ . \qquad (7)$$

where **(k + 1 l k)* is a priori estimator and *5t(k + llk + 1)* is a posteriori estimator. *F* is the Kalman filter coefficient with the system parameters in Eq. (3).

From the kinematic Kalman filter equation, the estimate of the speed of the vehicle is

$$v(k) = [0\ 1]\ X(k)\ . \qquad (8)$$

2.2 Orientation Estimation

Suppose that the orientation angle measured by the magnetometers in an IMU is

$$Omag(k) = OT (k) + Em_{ag}(k)\ . \qquad (9)$$

where Om_{ag} *(k)* is the magnetometer measurement, $_{4T}$ *(k)* is the true orientation, and $Em_{ag}(k)$ is zero-mean white Gaussian noise with the covariance of A_{Mag}. From the geometrical representation, the orientation of a vehicle driving at the speed of v(k) with the steering angle of *6(k)* can also be defined as

$$cb(k + lik) = ckklk) + \frac{7}{t\,vehicle} v(k) \tan 0\ (k)\ . \qquad (10)$$

where *7(k + llk)* is a priori estimate of the orientation angle, and *(k)* is the estimate of the vehicle speed by the kinematic Kalman filter. Since Eqs. (9)

and (10) deal with the same physical quantities, they can be fused for a better estimate of the orientation angle, i.e.

$$c4(k + 1Ik \pm 1) = kk + lik) \pm L [(\phi_{m\,ag}(k \pm 1) — ikk + _{lik)1} . \quad (11)$$

where *L* is the scaled covariance from cbm_{ag} to *cb(k + 1* lk), i.e.

$$L = X\ ikk+ilk)itiGyr(k+11k)^{-} \overset{Ar\ —1}{-}\ JGyr(k+ilkoGyr(k+iik) \quad (12)$$

Since $.i.ma_g\ (k +$ **ilk)** is $^{-}(4.(k + llk) + E\ m\ a_g(k),\ ^{-X,}Rk+ilk)^{-}4 - 6\ ma,\ (k+^{1}1\ k)$ *is*

$$X_{(k+^{1}1k)i4Mag}\ (k+i\ lk) — {}^{\cdot}q^{ii}(^{k}\ 1\ k)F(k\ I\ k) + \frac{T^2}{'vehicle}2(^{1}4A0 + 03 -Ay + AB\ 11_{,,}) . \quad (13)$$

Similarly, $X_{-,,,,,,}\ (k+1\ 1\ k)_{,,,,,,,}\ (k+1\ 1\ k)$ is

$$X_{omag(k\pm iik),}i_{;mag}\ (_{k\pm i}l_k) — X_{o(k\pm 1ik)}i^{-}{}_{km,t9\ (k\pm iik)} \pm Am_{ag} . \quad (14)$$

Notice that Eqs. (13) and (14) are time-varying quantities. Thus, the update gain, *L* in Eq. (12), is also time-varying and is to be calculated every sampling instance.

2.3 Localization by Sensor Fusion

For a vehicle driving at the speed of v(k) with the steering angle of *O(k),* the location at the next step can be predicted from the current location, i.e.

$$to(k + llk) = O(14) + 11(k1^{k)}[_{p(k)((1)}\ ^{\sin}\ cos^{(k)}\ _{a(k))]} \cdot \quad (15)$$

where *p(k)* is the radius of curvature, *a(k)* is the change of the orientation (i.e., *a(k) = O(k + 1) — O(k)),* and

$$p(k) = {}_{r}x(k)]\ ,\ R2 \quad [y() \quad \text{and } R(k) = \begin{bmatrix} \cos q5(k)1 \\ \sin O(k) \cos O(k) \end{bmatrix} \in R\ x\ 2\ 2 . \quad (16)$$

where *x(k)* and y(k) are the latitude and longitude of the vehicle in the absolute coordinate system. Since the sampling period, T, is very small, it is reasonable to assume that *a(k)* is small. Therefore, cos *a(k)* and sin *a(k)* can be approximated respectively to 1 and *a(k),* and thus Eq. (15) can be reduced to

$$p(k + llk) = 0(14) + 5(k)f)(k)T . \quad (17)$$

where *f;(k)T = p(k)a(k)* by the kinematic relationship, and

o , , = [s i n

$$\theta(k)]$$

$$\sin\theta(k) \qquad (18)$$

On the other hand, the latitude and longitude of the vehicle can also be measured by a GPS. Suppose that the measurement of the GPS is

$$PGPs(k) = \mathrm{PT}(k) + EGPS(k) \bullet \qquad (19)$$

where *pGps(k)* is the GPS measurement, *pT(k)* is the true location, and *EGps(k)* is a zero-mean white Gaussian noise with the covariance of *AGps*. The GPS measurement can be utilized to update the prediction by Eq. (17), i.e.

$$+ 1 \, lk + 1) = to(k + 11k) + [PGPs(k + {}^{1}) — 1^{5}(k + {}^{1}1k)i \, . \qquad (20)$$

where *J* is the scaled covariance from *pGps(k + 1)* to lio(k *+11k)*. Assuming that the latitude and longitude measurements are uncorrelated, *J* is defined as

$$J — \mathrm{diag} \quad \begin{matrix} X\text{-}1 \, {}^{X}Vk+{}^{1}1^{1}0\pm GPs(k+lik)`'i\cdot GPS(k\text{-}Filk)Gps(k+11k)' \\ {}^{A}xk+{}^{\prime}1k)DGps(k+{}^{\prime}1k);G^{\prime}Ps(k\text{-}Filk)Gps(k+1.1k) \end{matrix} \quad \mathrm{E} \; R^{2x2} \qquad (21)$$

where $^{-}\&Gps(k +11k) = {}^{-}\pm'(k +11k) + EGps(k)$ by simple calculations. Therefore, the first element of *J is*

$$Xl\cdot(k+iik)Gps(k+ilk) = (k1k)g \; (kik)$$
$$+T^{2} \; (A \cos^{t} OT \; v3\text{-},A_{ck} \sin^{2} \qquad A_{v}A_{o} \sin^{2} cbT) \qquad (22)$$

Similarly,

$$Xicps(k+11k)Gps(k\text{-}Flik) = X''\&(k\text{-}F1110''_{G}p_{s}(k\text{-}1\text{-}11k) + AGPS \bullet \qquad (23)$$

and

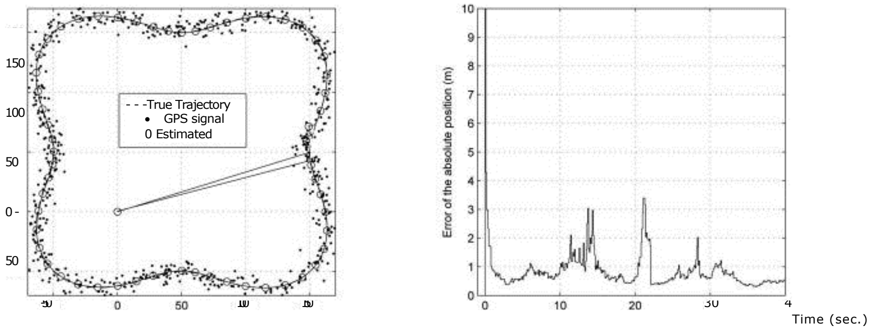
$$XVk\text{-}klik)Gps(k+11k) — fi(k1^{1}4^{T}(klk) \qquad (24)$$
$$+T^{2} \; (A, \sin^{2} OT + 4110 \; COS^{2} \; OT \, A_{v}A_{c}k \; COS^{2} \; cbT)$$

$$\text{-}X\text{-}Gps(k+11k)PGps(k+11k) = XVk+1110gGps(k+111c) \; AGPS \bullet \qquad (25)$$

Notice that *J* in Eq. (21) is time-varying, which results in a time-varying estimation algorithm.

3 Simulation Study

Figure la shows a true absolute position, an estimated absolute position by a GPS, and an estimated absolute position by the proposed sensor fusion algorithm. The estimated absolute position by the GPS shows a random error of ±5m with respect to the true absolute position, and the estimated absolute position by the proposed sensor fusion algorithm is close to the true absolute position after the transient response, which is because the initial position in the algorithm was set to (0,0), while the true position was (150,50). Fig. lb shows an estimation error of the proposed sensor fusion algorithm. It should be noted that the magnitude of estimated error was reduced to 4m/s in lsecond. Also, the increase of the estimated error is shown during simulation. This is because the predicted absolute position is not properly corrected in the interval where the GPS signal shows a large error such as near the point (150, 150).



(a) Estimation of the absolute position.

(b) Estimation error of the absolute position.

Fig. 1: Vehicle trajectory and estimation error.

4 Summary

In this paper, an advanced sensor fusion algorithm based on the extended Kalman filtering for the estimation of the absolute position (i.e., the latitude and longitude), speed, and orientation of a self-driving vehicle was proposed. The extended Kalman filter integrated an IMU, an encoder, a potentiometer with a GPS. The simulation result verified that the proposed sensor fusion algorithm successfully estimated the desired physical quantities in real-time.

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